
Mathematical Algorithms for Medical Imaging
Algorithmes mathématiques pour l'imagerie médicale
(Org: **Sima Noghanian** (Manitoba))

JORGE ALPUCHE, Department of Physics & Astronomy, University of Manitoba, Winnipeg, MB
The Application of the Filtered Back Projection (FBP) Algorithm to Quantitative Scatter CT Reconstruction

Introduction: Computed Tomography (CT) uses FBP to acquire Linear Attenuation Coefficient (LAC) images at a specific energy using transmitted photons. In typical CT studies a large proportion of photons are scattered and can lead to a reduction in contrast and a decrease in signal to noise ratio. However these photons carry valuable material information and this work presents a technique which uses a variant of the FBP algorithm to reconstruct images of Electron Density (ED) from scattered photons.

Methods: In the absence of attenuation the total number of scattered photons is given by the integral of ED along a narrow strip of material (eq. (1)).

$$N_s(E_0) = N_0(E_0) e\sigma(E_0) \int \rho_e(x) dx \quad (1)$$

where $N_s(E_0)$ is the number of scattered photons resulting from N_0 incident photons of energy E_0 , $e\sigma(E_0)$ is the probability of scattering for a photon of energy E_0 and $\rho_e(x)$ is the electron density at point x . In conventional CT the FBP algorithm is used to reconstruct ray integrals of the LAC. Similarly, our Scatter CT system reconstructs images of ED using FBP. Our system was simulated both with and without attenuation and Attenuation Correction Factors (ACFs), which were applied iteratively, were developed to correct for the attenuation.

Results and Conclusions: Attenuation free simulations reconstructed using the FBP yielded EDs with errors ranging from -0.5% to -2.1%. Simulations which included attenuation were reconstructed using the correct ACFs and yielded EDs with errors ranging from -2.7% to 1.1% after six iterations. These results show that under appropriate attenuation corrections, Scatter CT is capable of quantifying EDs assuming mono-energetic beam conditions and an accurate rejection of multiple scatter.

ALI ASHTARI, University of Manitoba
Resolution limits in nonlinear microwave image reconstruction

The resolution of microwave imaging methods based on linear radar signal processing such as synthetic aperture radar (SAR) is well known [1] and is directly dependent on the radar signal bandwidth. There are also studies about the resolution of other inverse scattering methods such as diffraction tomography, Born iterative method and distorted Born iterative method [2]. However, to the best of our knowledge, there has been no study on the resolution of optimization based inverse scattering methods. In this presentation, we emphasize on the differences between the optimization based methods and other inverse scattering methods in terms of the resolution. Preliminary observations on the possibility of having very high resolution images using optimization based methods are shown. Also, different natures of the SAR imaging and optimization based imaging are highlighted so that the major difference in the dependability of the resolution on bandwidth in these two methods is justified.

References

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- [2] T. Cui, W. C. Chew, X. X. Yin and W. Hong, *Study of resolution and super resolution in electromagnetic imaging for half-space problems*. IEEE Trans. Antennas Propagation **52**(2004), 1398–1411.

HOMA FASHANDI, University of Manitoba
Frequency Based Portal Image Registration for Radiotherapy Treatment

In External Beam Radiotherapy (EBRT), one cancer treatment method, external source of radiation is directed at the tumor from an external source produced by linear accelerator (LINAC). EBRT consists of two major parts; planning and treatment phases. In the planning phase, the shape and location of the tumor is determined by a simulator and in the treatment phase high energy beams irradiate the tumor. Reducing patient positioning uncertainty for each fraction of treatment process is crucial for both cancerous and healthy cells. To reduce geometric error, increasing the frequency of treatment verification with portal imaging would be an effective method. Portal images are taken with linear accelerator devices with therapeutic beams. Patient positioning problem could be considered as an image registration problem between the images taken during the planning phase and the ones taken during each treatment fraction (portal images). Images taken by therapeutic beams have low resolution and contrast. Limited contrast adaptive histogram equalization is used to enhance their quality. The major focuses of our method are log-polar and Fourier transforms properties.

Log-polar transformation converts scaling and rotation into translation; so we can deal with any angle of rotation and large scaling in the log-polar domain. To recover the parameters of the affine transformation, we map frequency domain to the log polar space and as an optimization method the normalized cross power spectrum of mapped portal and reference image is calculated. The scale and rotation will be recovered and phase correlation method will be used for determining translation.

DANIEL FLORES, University of Manitoba
Wavefront Reconstruction Method for Microwave Imaging

In recent years, radar technology has started to being used in a wide range of subsurface imaging applications. Traditionally, linear scan trajectories were used to acquire data in most of the subsurface radar applications. However, novel applications, such as Breast Microwave Imaging, require the use of non linear scan trajectories in order to perform their data acquisition process. This paper proposes a novel reconstruction algorithm for subsurface radar data acquired along quasi-elliptical and circular trajectories. The spectrum of the collected data is processed in order to locate the spatial origin of the target reflections and remove the diffraction artifacts introduced by the scan trajectory. The effects of the antenna mainlobe beamwidth on the quality of the reconstructed images is discussed and illustrated. The proposed algorithm was tested using simulated examples and data collected from phantoms that mimic breast and cancer tissue.

MICKAEL GERMAIN, Université de Montréal, CRM, Montréal
Lie algebra data processing applied to medical images

We propose to describe and develop a data processing technique that uses a new type of discrete transforms based on the orbit functions of compact Lie groups on the 3-dimensional (3D) case. These discrete orbit-function transforms (DOFT) are, in the particular case of a rectangular lattice of dimension $n = 2$ and of the Lie group $SU(2) \times SU(2)$, reduced to the transform known as 2-dimensional (2D) Discrete Cosine Transform Type-I. However, the DOFTs are unique as they are fast and allow processing of images defined on grids in the form lattices of other symmetries. A crucial property of the DOFTs is that it allows construction of continuous functions (trigonometric polynomials) providing continuous extension of the (inverse) transform from the discrete grid points to any point of the surface in between. It was demonstrated that the continuous extensions of DOFTs (CEDOFT) have superior analytic properties, such as convergence, localization, and differentiability of the trigonometric series when compared to other techniques. These properties (fast processing and continuous extension) suggest that these tools are ideal for interpolation processing. The proposal is aimed at using this process to interpolate data extracted from a MRI system. Compared to standard interpolation processes (tricubic, spline, etc.), we increase the quality of the interpolation, and the time computation is faster.

COLIN GILMORE, University of Manitoba, Winnipeg, Manitoba, Canada
An Overview of The Electromagnetic Inverse Problem with Biomedical Applications

Electromagnetic inversion and imaging as applied to the problems associated with biomedical imaging are described and reviewed. The mathematics of the electromagnetic inverse scattering problem will be outlined, as well as the basic problems of non-linearity, ill-posedness and non-uniqueness. Different inversion methods, such as linear and non-linear optimization will be discussed.

In particular, we will consider a non-linear inversion algorithm known as the Contrast Source Inversion (CSI) method. The CSI method formulates the inversion problem as an optimization problem which allows for non-constant velocities within the biological tissue and take into account multiple scattering. While slightly more mathematically complicated than linear inversion methods, these algorithms offer the possibility of reconstructing the quantitative material parameter values, such as permittivity and conductivity, within the biological material.

Computational results for 2D breast models are presented and show that the CSI algorithm provides a promising technique for biomedical imaging.

IAN JEFFREY, University of Manitoba
Adaptive Basis Functions Suitable for a Well-Conditioned Formulation to the Inverse Electromagnetic Scattering Problem under the BIM

The work presented shows that through an adaptive set of basis functions, the MoM solution to the linearized scalar inverse electromagnetic scattering problem is capable of alleviating the ill-conditioning of the resulting matrix equation. The selected basis functions, whole-domain and harmonic, provide a perfectly conditioned system of equations under the first-order Born approximation when appropriately selected field frequencies are chosen.

By analogy, we iteratively solve the full nonlinear problem using the Born Iterative Method (BIM) by introducing variability in the basis function expansion through a single phase parameter. By selecting the parameter value that minimizes the condition number of the discrete matrix operator, we demonstrate that it is possible to maintain a well-conditioned, linearized inverse problem, at each iteration of the BIM. The benefit of this approach is that it removes the requirement for Tikhonov regularization (or equivalent regularization schemes) usually needed to obtain physically meaningful solutions to the discrete system at each iteration.

AMIR MEGHDADI, University of Manitoba, Department of Electrical and Computer Engineering
Temporal and Spatial Imaging of Brain Epileptic Activities by Dynamical Characterization of EEG Signal

EEG signals may be used for detecting abnormal brain activities including epileptic seizures. Nonlinear time series analysis methods employ a dynamical system approach in order to better characterize the EEG signal. These methods assume a deterministic though very complex nature for the time series of EEG signal.

EEG signal during and possibly just before seizure activity is shown to be more deterministic. Detecting such determinism however is a challenging task because the signals are usually affected by noise. A new method for detecting determinism is proposed here which is robust to measurement noise and provides a tool for characterization of epileptic brain signals and locating the areas which are responsible for seizure generation in the brain.

PUYAN MOJABI, University of Manitoba, Electrical and Computer Engineering
The use of the L-curve and NCP parameter-choice methods in electromagnetic inverse scattering problems

It is well-known that the inverse scattering problem is inherently ill-posed: the solution is not unique and does not depend continuously on the data. For solving this ill-posed problem, we use Tikhonov regularization, which can be formulated as

a damped least squares problem, in conjunction with a parameter-choice method for finding the optimum regularization parameter. Finding the optimum regularization parameter is very difficult and also computationally expensive because the resulting solution can be very sensitive to the choice of the regularization parameter. Many regularization parameter-choice methods have been proposed in the literature: for example, generalized discrepancy principle, generalized cross validation, the L-curve and Normalized Cumulative Periodogram (NCP) method. The L-curve method tries to balance the (semi) norm of the solution and the corresponding residual by choosing the regularization parameter that puts one on the corner of the L-curve. The NCP method tries to use more available information from the residual as opposed to just the norm of the residual and it is based on the fact that there is similarity between the SVD basis and Fourier basis.

Herein, the application of the L-curve and NCP parameter-choice methods to the Tikhonov-regularized functional arising in the 2-D/TM inverse scattering problem which is formulated via an integral equation and solving using the Born iterative method (BIM) is investigated and adapted for this application.

MALCOLM NG MOU KEHN, University of Manitoba

Accuracy Improvement of an Existing Permittivity Measurement Technique for Dielectric Disk Samples

A method for permittivity measurement is re-studied. Though being able to determine the dielectric constant of disk samples, it suffers from frequency variation that can lead to errors as severe as 20% at certain frequencies. A technique that improves the accuracy is proposed here. It capitalizes on trends in the slopes of retrieved permittivity vs. frequency graphs to sieve out the undesired frequency dependence. These slope phenomena are characterized via numerical simulations of the measurement structure, which comprises a dielectric disk sample sandwiched between disjointed inner conductors of a coaxial cavity. Read-up data graphs for general usage are then obtained. Repetitive generation of such graphs is thus not necessary. With error levels of less than 1%, the accuracy of this improved method is significantly higher than that obtained by just directly applying the original technique alone. Being independent of the required reference materials, the method is also shown to be stable. In addition, an independently new technique for measuring the permittivity of annular ring samples using quadratic curve fitting is proposed. By measuring only three known materials (one of them may be free space, thus reducing only to two required known solid dielectrics), the permittivity of any unknown dielectric may subsequently be determined with high accuracy over a wide frequency range. Comparison results of accuracy between this new approach and the improved method mentioned earlier will be presented.

BARBARA PAWLAK, University of Manitoba

PET Image Reconstruction by Density Estimation

PET (positron emission tomography) scans are still in the experimental phase, as one of the newest breast cancer diagnostic techniques. It is becoming the new standard in neurology, oncology and cardiology. PET, like other nuclear medicine diagnostic and treatment technique involves the use of radiation. Because of negative impact of radioactivity to our body the radiation doses in PET should be small.

The existing computing algorithms for PET can be divided into two broad categories: analytical and iterative methods. In the analytical approach the relation between the picture and its projections is expressed by a set of integral equations which are then solved analytically. Iterative methods can be further divided into deterministic and stochastic approaches. The ART (Algebraic Reconstructed Technique) algorithm, developed and first used by Gordon, et al., in the reconstruction of biological material in early 1970s, is an example of deterministic technique. The stochastic approach, like EM (expectation maximization) algorithm, bases on the assumption that radioactive emissions follow Poisson statistics. The algorithm combines unique and properties of the Poisson process and the maximum likelihood method of estimation.

The proposed kernel density estimation algorithm falls also into the category of iterative methods. In this approach each coincidence event is considered individually. The estimate location of the annihilation event that caused each coincidence event bases on the previously assigned location of events processed earlier. To accomplish this, we construct a probability distribution along each coincidence line. This is generated from previous annihilation points by density estimation. It has been

observed that density estimation approach to PET can reconstruct an image of the existing tumor using significantly lower data than the standard CT algorithms, like Fourier back-projection. Therefore, it might be a very promising technique allowing to reduce the radiation dose for patients.

ABAS SABOUNI, University of Manitoba, Winnipeg, Manitoba, Canada, R3T 5V6
Parallel FDTD/GA for Microwave Image Reconstruction

In the past few years microwave imaging has received significant interest due to its potential to detect breast tumors at an early stage. Microwave imaging is the process by which radiofrequency electromagnetic waves are used to generate an image of a body to enable physicians to diagnose disease. To create images from microwave measurements, it is necessary to construct an electromagnetic field, which is able to transmit microwaves and measure the scattered waves at one or more sampling points. "Tomography" is one of the methods used in microwave imaging. In this method, to solve an inverse problem, a forward solver and an optimization tool are needed. The numerical Finite Difference Time Domain (FDTD) method is a powerful tool used as forward solver for solving Maxwell's equations to compute the scattered electric field at the observation points. The Genetic Algorithm (GA) is a popular evolutionary global optimization method that performs very well for problems with a high number of parameters and high degrees of non-linearity.

This talk presents an effective method of microwave imaging using FDTD as forward solver and GA for optimization. Since both FDTD and GA are computationally intensive parallel computations of the GA and FDTD codes are proposed. Using Message-Passing Interface (MPI) libraries, we are able to reach high-quality images with a reasonable run-time. The parallelization for the GA is based on master/slave protocol and for FDTD based on the distributed heartbeat algorithm. To the best of our knowledge, this implementation of hybrid method of parallel GA and FDTD represent a novelty in the framework of the finding early stage breast cancer application.

With contributions from Prof. Sima Noghianian and Prof. Stephen Pistorius (both University of Manitoba).

PARIMALA THULASIRAMAN, Univ. of Manitoba, Department of Computer Science, Winnipeg, MB, R3T 2N2
Design, development and implementation of a parallel algorithm for computed tomography using algebraic reconstruction technique

In this work, we examine the design and implementation of a parallel algorithm for reconstructing images from projections using the algebraic reconstruction technique (ART). This technique for reconstructing pictures from projections is useful for applications such as Computed Tomography (CT or CAT). The algorithm requires fewer views, and hence less radiation, to produce an image of comparable or better quality. However, the approach is not widely used because of its computationally intensive nature in comparison with rival technologies. A faster ART algorithm could reduce the amount of radiation needed for CT imaging by producing a better image with fewer projections.

A reconstruction from projections version of the ART algorithm for two dimensions was first designed for a distributed memory machine and implemented in parallel using the Message Passing Interface (MPI). The results produced on the distributed memory machine did not produce faster reconstructions due to prohibitively long and variant communication latency. The algorithm was then redesigned for a multithreaded, shared memory machine and implemented in OpenMP. This version produced positive results, showing a clear computational advantage for multiple processors and measured efficiency ranging from 60–95%. Consistent with the literature, image quality proved to be significantly better compared to the industry standard Filtered Backprojection algorithm especially when reconstructing from fewer projection angles.

NIRANJAN VENUGOPAL, University of Manitoba
A 3D Model-Based Outer Volume Suppression Technique for MR Spectroscopic Imaging of the Prostate

Purpose: To adapt a new MR Spectroscopy (MRS) technique employing non-cuboidal voxels, called conformal voxel MRS

(CV-MRS), for use in prostate spectroscopic imaging in order to reduce contamination of spectra by lipid signal surrounding the prostate.

Method and Materials: CV-MRS uses twenty or more spatial saturation (SS) pulses, placed around the prostate, to reduce the lipid signal affecting the spectra within the prostate. Use of the new CV-MRS technique reduced the lipid signal contamination by 84% as compared to standard cuboidal voxel MRS. To further reduce the lipid contamination, the routinely used 90 degree flip angle used for each SS pulse was modified to take into account the regrowth of lipid signal with its short T1 relaxation time.

Results: Resulting spectra from the optimized approach actually showed an increase in lipid contamination by 10%. We tracked the problem down to overlapping SS pulses. Using a simulated 3D model, we found that 68% of the volume we were trying to saturate experienced multiple overlapping SS pulses. Regions of the volume experiencing an even number of SS pulses were found to increase the lipid contamination signal by 88% to 200%. Conversely, regions experiencing an odd number of SS pulses had a reduction in lipid contamination of 55%.

Conclusion: Changing the ordering of the SS pulses, such that the overlapping pulses occur later in the train of 20 SS pulses reduces this problem. In summary, we have developed an improved outer volume saturation technique which reduces lipid contamination problems in prostate MRS.