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Partial unconditionality and Barriers

The purpose of this talk is to present a framework for studying weakly-null sequence of Banach spaces using the Ramsey property of the families of finite sets of integers called Barriers, introduced by Nash-Williams. We focus on "partial unconditionality" properties of weakly-null sequences of Banach spaces. Inspired by a recent work of S. J. Dilworth, E. Odell, Th. Schlumprecht and A. Zsak, we give a general notion of partial unconditionality that covers most of the known cases, including the classical Elton's near unconditionality, convex unconditionality or Schreier unconditionality, and some new ones.

The method reduces the problem to the understanding of mappings $\varphi\colon \mathcal{B} \to \mathrm{FIN} \times c_0$, where FIN denotes the family of finite sets, $\mathcal{B} \subseteq \mathrm{FIN}$ is a barrier, and c_0 is the Banach space of sequences of real numbers converging to zero. We present several combinatorial results concerning these mappings, starting with the simpler mappings $\varphi\colon \mathcal{B} \to \mathrm{FIN}$. One of the main results here is that every mapping $\varphi\colon \mathcal{B} \to c_0$ has a restriction which is, up to perturbation, what we call a L-mapping, *i.e.*, φ has a precise Lipschitz property and satisfies that the support $\operatorname{supp}\varphi(s)$ of $\varphi(s)$ is included in s for every $s \in \mathcal{B}$. L-mappings allow to define naturally a weakly-null sequence, called L-sequence, associated to them.

Finally our approach shows that if for some notion of unconditionality $\mathfrak{F}$ there is a weakly-null sequence with no $\mathfrak{F}$-unconditional subsequence, then there must be an L-sequence with no $\mathfrak{F}$-unconditional subsequence.

This is a joint work with S. Todorcevic.