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New Families of Integrable Diffusion Models and their Applications to Finance

We present some recent developments in the construction and classification of new families of analytically solvable one-dimensional diffusion models for which transition densities and other quantities that are fundamental to financial modeling and derivatives pricing are represented in closed form. Our approach allows us to uncover new multi-parameter processes that are mapped into various simpler diffusions. From an asymptotic analysis of the boundary behaviour of the processes, we arrive at a rigorous characterization of the newly constructed diffusions with respect to probability conservation and the martingale property. Specifically, we analyze in detail three subfamilies of models arising from the underlying squared Bessel process (Bessel family), the CIR process (confluent hypergeometric family) and the Jacobi diffusion (hypergeometric family). We show that the Bessel family is a superset of the constant elasticity of variance (CEV) model. The former, in turn, is generalized by the confluent hypergeometric family. For these two families we find further subfamilies of conservative strict super-martingales and absorbed martingales. For the new classes of absorbed diffusions we also derive analytically exact first-passage time densities, as well as probability densities for the extrema of the processes. Formulas are reduced to integral representations or eigenfunction expansions involving special functions. New closed-form pricing formulas for standard Europeans, barrier options and lookback options also follow. We conclude by discussing other mathematical finance applications and possible extensions of our models to include jumps and markov switching.