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Covering congruences

Over 50 years ago, Paul Erdős conjectured that the integers can be covered by a finite collection of residue classes $a_i \pmod{n_i}$ with distinct moduli n_i , and with the least modulus arbitrarily large. So far, the record is due to Morikawa in 1984, who has found such a covering system with least modulus 24. This, and similar problems, are discussed extensively in Guy's UPINT. We solve one of these problems, namely, we prove the conjecture of Erdős and Selfridge that in a covering system with large least modulus, the reciprocal sum of the moduli must also be large. In addition, we prove the conjecture of Erdős and Graham that for each $K > 1$, there is a positive number d_K such that if the moduli all come from an interval $[N, KN]$, where N is large, then for any choice of residue classes for these moduli, at least density d_K of the integers remain uncovered.

This work is joint with Michael Filaseta, Kevin Ford, Sergei Konyagin, and Gang Yu.