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Differential Categories

We introduce the notion of a *differential category*: a (semi-)additive symmetric monoidal category with a comonad (a "coalgebra modality") and a differential combinator, satisfying a number of coherence conditions. In such a category, one should regard the base maps as "linear", and the coKleisli maps as "smooth" (infinitely differentiable). Although such categories do not necessarily arise from models of linear logic, one should think of this as replacing the usual dichotomy of linear *vs.* stable maps established for coherence spaces.

To illustrate this approach, we give a number of examples, the most important of which, a monad S_∞ on the category of vector spaces, with a canonical differential combinator, fully captures the usual notion of derivatives of smooth maps. Our models are somewhat more general than are allowed by other approaches (such as Ehrhard's and Regnier's, which inspired our work). For example, differential categories are monoidal categories, rather than monoidal closed or $*$-autonomous categories. This allows us to capture various "standard models" of differentiation which are notably not closed. Second, we relax the condition that the comonad be a "storage" modality in the usual sense of linear logic, again so as to allow the standard models which do not necessarily give rise to a full storage modality. However, when the comonad is a storage modality, we can describe an extension of the notion of differential category which captures the not-necessarily-closed fragment of Ehrhard–Regnier's differential λ-calculus.

Joint work by R. Blute, J. R. B. Cockett, and R. A. G. Seely.