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Proper covers of left ample monoids

The relation \mathcal{R}^* is defined on a monoid M by the rule that $a\mathcal{R}^*b$ if and only if for all $x, y \in M$,

$$xa = ya \Leftrightarrow xb = yb.$$

If the set of idempotents $E(M)$ of M is a commutative submonoid of M and every \mathcal{R}^* -class contains an idempotent, M is said to be *left adequate*. In such a monoid each \mathcal{R}^* -class contains a unique idempotent and the idempotent in the \mathcal{R}^* -class of an element a is denoted by a^\dagger . A left adequate monoid M is *left ample* if $ae = (ae)^\dagger a$ for all $e \in E(M)$ and $a \in M$.

Thus a right cancellative monoid is left ample; here \mathcal{R}^* is the universal relation. Every inverse monoid is left ample.

A left ample monoid is *proper* if the intersection of the minimum left cancellative congruence and \mathcal{R}^* is trivial. The structure of proper left ample monoids can be described in terms of right cancellative monoids and commutative monoids of idempotents. Moreover, any left ample monoid M has a *proper cover*, that is, a proper left ample monoid P together with a homomorphism from P onto M which restricts to an isomorphism from $E(P)$ onto $E(M)$. We consider how such covers can be constructed.