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Semiclassicalisaton of quantum differentials and Poisson geometry

We semiclassicalise the standard notion of differential calculus in noncommutative geometry on algebras and quantum groups. We show in the symplectic case that the infinitesimal data for a differential calculus is a symplectic connection, and interpret its curvature as lowest order nonassociativity of the exterior algebra. In the Poisson–Lie group case we study left-covariant infinitesimal data in terms of partial connections. We show that the moduli space of bicovariant infinitesimal data for quasi-triangular Poisson–Lie groups has a canonical reference point which is flat in the triangular case. Using a theorem of Kostant, we completely determine the moduli space when the Lie algebra is simple: the canonical partial connection is the unique point for other than $\mathfrak{sl}_n$, $n > 2$, when the moduli space is 1-dimensional. This proves that the deformation-theoretic exterior algebra on standard quantum groups must be nonassociative and we provide it as a super-quasiHopf algebra. More generally, we show that many standard quantisations in physics including of coadjoint orbits (such as fuzzy spheres) have naturally nonassociative differential structures. Our methods also quantise quasi-Poisson manifolds of interest in string theory.

Mostly joint work with E. J. Beggs.