Representation Theory
La théorie des représentations
(Org: **Wentang Kuo** (Waterloo))

GERALD CLIFF, University of Alberta, Dept. of Math. & Stat. Sci., Edmonton, AB T6A 0M2
Realizing the local Weil representation over a number field

Let K be a nonarchimedian local field whose residue field has $q = p^m$ elements. Let W be the Weil representation of the symplectic group $\mathrm{Sp}(2n, K)$. We show that W, considered either as a projective representation of $\mathrm{Sp}(2n, K)$ or a representation of the metaplectic group $\mathrm{Mp}(2n, K)$, has a model defined over the field $Q[\sqrt{p}, \sqrt{-p}]$. We use the Schrödinger model of the Heisenberg group, having W act on locally constant, compactly supported complex functions $S(Y)$ on an n-dimensional K space Y. We replace $S(Y)$ by E-valued functions (still locally constant, of compact support) on Y, where E is the field obtained from Q by adjoining all p-power roots of unity. (This is in the case that the characteristic of K is 0.) Then we use Galois cohomology. As an application, we show that the local theta correspondence can also be defined for representations over a number field of a dual reductive pair.

This is joint work with David McNeilly, also of the Unversity of Alberta.

CLIFTON CUNNINGHAM, University of Calgary, Alberta, T2N 1N4
Depth-zero Character Sheaves

In this talk we describe the correspondence between characters of depth-zero supercuspidal representations of p-adic groups and coefficient systems on the Bruhat–Tits building of perverse sheaves. Simple objects in this category are called *depth-zero character sheaves*. We associated a distribution to each depth-zero character sheaf which can be used to recover character values. Although the correspondence does not match irreducible representations with depth-zero character sheaves, the distributions associated to depth-zero character sheaves appear naturally when studying L-packets of representations. We will also indicate how depth-zero character sheaves may be viewed as objects in a derived category of ℓ-adic sheaves on a rigid analytic space; this perspective suggests how to build sheaves for the characters of a more general class of admissible representations.

JULIA GORDON, University of Toronto
Characters of depth-zero representations and motives

Most characters of representations of p-adic groups elude explicit computation. In this talk, we will discuss an approach to this problem that is based on motivic integration, which allows us to attach geometric objects to all "natural" p-adic integrals. As a result, we will be able to show that the values of characters of depth-zero supercuspidal representations of most classical groups can be obtained by "counting points" of some geometric objects over the residue field.

KYU-KWAN LEE, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3
Spherical Hecke algebras of $\mathrm{GL}(n)$ over 2-dimensional local fields

The classical Satake isomorphism plays an important role in the Langlands program. In this talk we will try to generalize the theory to the 2-dimensional local field case. More precisely, we will construct the spherical Hecke algebra of $\mathrm{GL}(n)$ over a 2-dimensional local field, and try to define an analogue of the Satake isomorphism, using Fesenko's $R((x))$-valued measure. A connection to Kac–Moody groups will also be briefly discussed.

This is joint work with Henry Kim.

PAUL MEZO, Carleton University, 1125 Colonel By Drive, Ottawa, ON K1S 5B6
Automorphism-invariant representations of real reductive groups

There is a well-established classification of the irreducbible representations of a real reductive group in terms of discrete-series representations and their related data. Given an group automorphism, it is natural to wonder whether one can classify

the irreducible representations which are invariant under the automorphism in a similar manner. We shall provide such a classification in the case of split groups and give an indication towards applications and future generalizations.

FIONA MURNAGHAN, University of Toronto
Distinguished tame supercuspidal representations

Let θ be an involution of a group G. A representation of G is said to be θ-distinguished if there exists a nonzero linear functional on the space of the representation that is invariant under the fixed points G^θ of θ in G. Suppose that G is a connected reductive p-adic group. We discuss distinguishedness of tame supercuspidal representations of G, relative to various involutions. As an application, we can determine when two tame supercuspidal representations of G are equivalent, in terms of conditions on the associated cuspidal G-data used to construct the representations.

MONICA NEVINS, University of Ottawa, Ottawa, ON
Branching Rules for Principal Series of $\mathrm{GL}(3)$

Let k be a p-adic field with integer ring $\mathcal{R}$. We consider the restriction of a principal series representation of $\mathrm{GL}(3,k)$ to a maximal compact subgroup $K = \mathrm{GL}(3,\mathcal{R})$. Its decomposition into irreducible subrepresentations is perplexing and marvelous. This is an update on a joint work with Peter Campbell (University of Ottawa).

WULF ROSSMANN, University of Ottawa, Ottawa
Representations of $\mathrm{SL}(2,\mathbb{Z})$ *and elliptic modular functions*

The group $\mathrm{SL}(2,\mathbb{Z})$ admits remarkable representations on finite-dimensional spaces constructed from classical theta functions. It turns out that these representations can be viewed as oscillator representations of finite quotients $\mathrm{SL}(2,\mathbb{Z}/n\mathbb{Z})$. Particular examples of such groups are the binary polyhedral groups corresponding to Dynkin diagrams of type D_5, E_6, E_7, E_8 by the McKay correspondence. The purpose of the talk is to point out a striking analogy between these representations of $\mathrm{SL}(2,\mathbb{Z})$ and certain representations of Weyl groups on spaces of characters of semisimple Lie groups, the characters of the Lie group playing the role for the Weyl group which the modular elliptic functions play for $\mathrm{SL}(2,\mathbb{Z})$.

LOREN SPICE, University of Michigan, 430 Church St., Ann Arbor, MI
Supercuspidal characters of p*-adic* SL_ℓ*,* ℓ *a prime*

Character computations, and in particular supercuspidal character computations, are an important part of p-adic harmonic analysis. In this talk, we arrive at explicit supercuspidal character formulae for SL_ℓ over a p-adic field by evaluating an integral formula due to Harish-Chandra. Our computations also allow us to describe explicitly the local Langlands parameters of many supercuspidal representations of GL_ℓ.

FERNANDO SZECHTMAN, University of Regina, Regina, SK
The Steinberg lattice of a finite Chevalley group and its modular reduction

The talk will review a paper by R. Gow and touch upon a minor contribution by the speaker.

Let $G = G(\Phi, F_q)$ denote the finite Chevalley group associated to an indecomposable root system Φ over a finite field F_q of characteristic p. In 1957 R. Steinberg constructed a minimal left ideal I of the integral group algebra $\mathbf{Z}G$ possessing some remarkable properties. One of these is that I is a free $\mathbf{Z}$-module whose rank is the p-part of $|G|$; this gives rise to an integral matrix representation of G, which viewed as a complex representation is irreducible. Gow studies what happens to this matrix representation when it is reduced modulo a prime. Our contribution occurs when Φ is of type $\mathbf{C}_n$ and the reduction is modulo 2.

CHIAN-JEN WANG, University of Minnesota, 127 Vincent Hall, 206 Church St. S.E., Minneapolis, MN 55455
Distinguished representations of metaplectic groups

An automorphic representation on a metaplectic cover of GL is called "distinguished" if it has a unique Whittaker model. Distinguished representations can be viewed as generalizations of classical theta functions. Using the method of the converse

theorem, Patterson and Piatetski–Shapiro constructed cuspidal distinguished representations on the three-fold covers of $\mathrm{GL}(3)$. In this talk, we will discuss recent progress toward generalizing the work of Patterson and Piatetski–Shapiro to the case of four-fold covers of $\mathrm{GL}(4)$.

WAI LING YEE, University of Alberta
Signatures of Invariant Hermitian Forms

Classifying the irreducible unitary representations of a reductive Lie group may be formulated as the algebraic problem of classifying the irreducible Harish–Chandra modules which admit a positive definite invariant Hermitian form. It is thus of interest to study signatures of invariant Hermitian forms and to understand how positivity can fail. A special case, which may be a necessary first step in finding a general answer, is the computation of the signature of the Shapovalov form on irreducible Verma modules $M(\lambda)$. Computing the signature of the Shapovalov form on irreducible highest weight modules $L(\lambda)$ may provide insight into the potentially analogous problem of computing signatures of invariant Hermitian forms on standard limit representations and perhaps may yield some interesting information concerning composition series of Verma modules.

JIU-KANG YU, Purdue University
A construction of types

I will present a joint work with Julee Kim on a construction of types.

KAIMING ZHAO, Wilfrid Laurier University
Weight representations of higher rank Vorasoro algebras

In this talk, I will summarize results leading to the classification of irreducible weight modules with finite dimensional weight spaces over higher rank Virasoro algebra. The classification for such modules for rank one was given by O. Mathieu with a completely different approach in 1993.

This talk is based on a joint paper with R. Lu.