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Erdős–Hajnal Sets and Semigroup Decompositions

Define a set of lines in R^3 to be "stacked" with respect to $v \in R^3$ if, from a vantage point far away in the direction of v, the lines are linearly ordered by the "crossing over" relation. Given a collection of skew lines and a point v, we ask, what is the largest stacked subset that must be present among the lines? This question, which appears in a a 2000 paper of Erdős, Hajnal, and Pach, is intimately related to the well-known Erdős–Hajnal conjecture via the Milnor–Thom theorem. It was recently resolved by a powerful and very general theorem of Alon, Pach, Pinchasi, Radoicic, and Sharir.

We describe these results and discuss several related issues, including a generalization to "Erdős–Hajnal sets" and an intriguing problem concerning the decomposability of semi-algebraic sets: Do all semi-algebraic sets belong to the set algebra generated by semigroups in R^d? Our main result is a resolution of this question in dimensions 1 and 2.