This talk will present a few advances on some earlier work on the following question: when is a smooth real surface $S \subset \mathbb{C}^2$ locally polynomially convex at a point $p \in S$? This question is complicated by the presence of points in the surface $S$ that have complex tangents. Such points are called CR singularities. Let $p \in S$ be a CR singularity at which the order of contact of the tangent plane with $S$ is greater than 2; i.e., a degenerate CR singularity. We will discuss a sufficient condition for $S$ to be locally polynomially convex at a degenerate CR singularity. In demonstrating this sufficient condition, we will need a new result about the uniform algebra on a closed disc in $\mathbb{C}$ generated by $z$ and a complex-valued continuous function $F$. This result may be of independent interest because the function $F$ here is allowed to be non-smooth; this result may thus be viewed as a Mergelyan-type theorem for complex-valued $F$. 