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Exterior Monge–Ampère solutions for real convex bodies

We study solutions of the homogeneous complex Monge–Ampère equation $(\partial\bar{\partial}u)^n = 0$ in $\mathbb{C}^n \setminus K$, where K is a compact convex set in $\mathbb{R}^n \subset \mathbb{C}^n$ for a plurisubharmonic function growing logarithmically on $\mathbb{C}^n$ and tending to zero along K. The solution we are considering is therefore the Siciak–Zaharjuta extremal function associated to K. We use the method of characteristics to describe the solution. We describe a variational problem for Robin constants associated to holomorphic disks passing through the hyperplane at infinity in $\mathbb{CP}^n$, which is in a sense dual to the Kobayashi–Royden functional on disks used to define the infinitesimal Kobayashi distance. We use Lempert's work along these lines for the exteriors of strictly convex sets in $\mathbb{C}^n$, passing to a limit of a sequence of approximating domains. The characteristic curves of the extremal function for the limit real convex set always exist and we show they are given by quadrics in $\mathbb{CP}^n$. The variational formulation and elementary geometry enable us to analyze the extremal functions fairly explicitly, especially under weak regularity conditions on K. An application is given to polynomial approximation in higher dimensions.

This is joint work with Norman Levenberg (Indiana University) and Sione M'au (Aukland University).