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*Hybrid Symbolic-Numeric Integration in Multiple Dimensions via Tensor Product Series*

We present a new hybrid symbolic-numeric method for the fast and accurate evaluation of definite integrals in multiple dimensions. This method is well-suited for two classes of problems:

- (1) analytic integrands over general regions in two dimensions, and
- (2) families of analytic integrands with special algebraic structure over hyperrectangular regions in higher dimensions.

The algebraic theory of multivariate interpolation via natural tensor product series was developed in the doctoral thesis by Chapman, who named this broad new scheme of bilinear series expansions “Geddes series” in honour of his thesis supervisor. This talk describes an efficient adaptive algorithm for generating bilinear series of Geddes–Newton type and explores applications of this algorithm to multiple integration. We will present test results demonstrating that our new adaptive integration algorithm is effective both in high dimensions and with high accuracy. For example, Carvajal’s Maple implementation of our algorithm has successfully computed nontrivial integrals with hundreds of dimensions to 10-digit accuracy, each in under 3 minutes on a desktop computer.

Current numerical multiple integration methods either become very slow or yield only low accuracy in high dimensions, due to the necessity to sample the integrand at a very large number of points. Our approach overcomes this difficulty by using a Geddes–Newton series with a modest number of terms to construct an accurate tensor-product approximation of the integrand. The partial separation of variables achieved in this way reduces the original integral to a manageable bilinear combination of integrals of essentially half the original dimension. We continue halving the dimensions recursively until obtaining one-dimensional integrals, which are then computed by standard numeric or symbolic techniques.

This talk presents joint research with Orlando Carvajal and Keith Geddes.