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Substitutions, numeration and tilings

The connections between substitutions and numeration systems are numerous and natural: in particular, one can define a numeration system based on finite factors of an infinite word generated by a primitive substitution σ, known as the Dumont–Thomas numeration; this numeration system provides generalized radix expansions of real numbers with digits in a finite subset of the number field $\mathbb{Q}(\beta)$, β being the Perron–Frobenius eigenvalue of σ. When σ is a substitution of constant length l, one recovers the classic l-adic numeration. A characteristic example is given by the Fibonacci substitution $1 \mapsto 12$, $2 \mapsto 1$ and by the Fibonacci numeration (where non-negative integers are represented thanks to the usual Fibonacci recurrence with digits in $\{0, 1\}$ and no two one's in a row allowed).

It is possible, generalizing Rauzy's and Thurston's constructions, to associate in a natural way either with a Pisot number β (of degree d) or with a Pisot substitution σ (on d letters) some compact basic tiles that are the closure of their interior, that have non-zero measure and a fractal boundary. We know that some translates of these prototiles under a Delone set Γ (provided by β or σ) cover $\mathbb{R}^{d-1}$; it is conjectured that this multiple tiling is indeed a tiling (which might be either periodic or self-replicating according to the translation set Γ). This conjecture is known as the Pisot conjecture.

The aim of this lecture is to state for Pisot substitutions a finiteness property in terms of the Dumont–Thomas numeration which is a sufficient condition to get a tiling.

This is joint work with A. Siegel.