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Fluid flow in the semigeostrophic oceans and atmosphere

The semigeostrophic approximation to Euler's equation is used to caricature the large-scale, long-time evolution of the atmosphere and oceans, and to model such phenomena as the formation and evolution of pressure fronts. Even in 3D, it can be formulated as an active scalar transport model—like the 2D incompressible Euler equations—but with the Hessian of the stream function related to the conserved scalar quantity by a Determinant instead of a Trace. This talk surveys some mathematical developments and open questions concerning this nonlinear model, exposing in particular a family of exact solutions to the equations, representing 2D circulations of an ideal fluid in a elliptical ocean basin. For these special solutions, the fluid pressure and stream function remain quadratic functions of space at each instant in time, whose fluctuations are described by a single degree of freedom Hamiltonian system depending on two conserved parameters: domain eccentricity and the constant value of potential vorticity. These parameters determine the presence or absence of periodic orbits with arbitrarily long periods, fixed points of the dynamics, and aperiodic homoclinic orbits linking hyperbolic saddle points. The energy relative to these parameters selects the frequency and direction in which isobars nutate or precess, as well as the steady circulation direction of the fluctuating flow. Canonically conjugate variables are given, which describing the complete evolution of an elliptical inverse-potential-vorticity patch in dual space.