Representations of Algebras
Représentations des algèbres
(Org: **Ibrahim Assem**, **Thomas Brustle** and/et **Shiping Liu** (Sherbrooke))

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Generic complexes and derived representation type for Artin algebras

Let A be an Artin algebra and $D(A)$ its bounded derived category. We recall that $D(A)$ is called discrete and A derived discrete if for any sequence $h = (h_i)_{i \in \mathbb{Z}}$ of non-negative integers with almost all the $h_i = 0$, there are only finitely many isoclasses of indecomposable objects $X \in D(A)$ with length of $H^i(X) = h_i$ for all $i \in \mathbb{Z}$.

We prove the following:

Theorem *The Artin algebra A is not derived discrete if and only if there is a bounded complex of projective A-modules $X = (X^i, d_X^i)$ with the following properties:*

(i) for all i the image of d_X^i is in the radical of X^{i+1};

(ii) X is indecomposable in the homotopy category of complexes;

(iii) there is some j such that $H^j(X)$ has not finite length;

(iv) for all i, $H^i(X)$ has finite length as left E-module, where E is the endomorphism ring of X in the homotopy category of complexes.

A complex as before is called generic complex. In case A is a finite-dimensional algebra over an algebraically close field, we also consider the tame representation type in terms of generic complexes.

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Universal deformation rings and dihedral defect groups

Let k be an algebraically closed field of characteristic 2, and let W be the ring of infinite Witt vectors over k. Suppose G is a finite group, and B is a block of kG with dihedral defect group D which is Morita equivalent to the principal 2-modular block of a finite simple group. We determine the universal deformation ring $R(G, V)$ for every kG-module V which belongs to B and has stable endomorphism ring k. It follows that $R(G, V)$ is always isomorphic to a subquotient ring of WD. Moreover, we obtain an infinite series of examples of universal deformation rings which are not complete intersections.

WALTER BURGESS, Department of Mathematics and Statistics, University of Ottawa, Ottawa, ON, K1N 6N5

On the quasi-stratified algebras of Liu and Paquette

Quasi-stratified algebras are a generalization of standardly stratified algebras for which both the Cartan Determinant Conjecture and its converse hold. This talk will relate quasi-stratified algebras to other classes of algebras: left serial, the Yamagata algebras and gentle algebras.

Joint work with Ahmad Mojiri.

XUEQING CHEN, University of Wisconsin–Whitewater
Root Vectors, PBW and Canonical Bases of Ringel–Hall Algebras and Quantum Groups

Let $\mathbf{g} = \mathbf{g}(C)$ be the Kac–Moody Lie algebra associated to a Cartan matrix C and $\mathbf{U} = \mathbf{U}_v(\mathbf{g})$ its quantum group. A key feature in quantum groups is the presence of several natural bases (like the PBW-basis and the canonical basis). There are different approaches to the construction of the canonical basis: algebraic approach (Lusztig, Kashiwara, Beck–Chari–Pressley, Beck–Nakajima), geometric approach (Lusztig) and Ringel–Hall algebra approach (Ringel, Lin–Xiao–Zhang). In this talk, we will recall algebraic and Ringel–Hall algebra approaches to a PBW basis and a canonical basis of $\mathbf{U}$ when C is finite or affine. Meanwhile, the root vectors in Ringel–Hall algebras will be discussed.

FLAVIO ULHOA COELHO, University of São Paulo, Departamento de Matemática–IME, Rua do Matão, 1010, Cidade Universitária/CEP, 05508-090 São Paulo, Brazil
On the composition of irreducible morphisms

Let A be an artin algebra. Using the so-called Auslander–Reiten theory, one can assign to A a quiver Γ_A called the *Auslander–Reiten quiver of A* which "represents" the indecomposable finitely generated A-modules together with some morphisms between them called irreducible. Unfortunately, Γ_A does not give all the informations on the category $\operatorname{mod} A$ of the finitely generated A-modules one could expect because not all morphisms can be re-constructed from the irreducible ones. However, (sum of) compositions of irreducible morphisms can give important informations on $\operatorname{mod} A$.

A morphism $f\colon X \to Y$ is called *irreducible* provided it does not split and whenever $f = gh$, then either h is a split monomorphism or g is a split epimorphism. It is not difficult to see that such an irreducible morphism f belongs to the radical $\operatorname{rad}(X, Y)$ but not to its square $\operatorname{rad}^2(X, Y)$. Consider now a non-zero composition $g = f_n \cdots f_1\colon X_0 \to X_n$ of $n \geq 2$ irreducible morphisms $f_i's$. It is not always true that $g \in \operatorname{rad}^n(X_0, X_n) \setminus \operatorname{rad}^{n+1}(X_0, X_n)$. In this talk, we shall discuss some results on the problem of when such a composition does lie in $\operatorname{rad}^n(X_0, X_n) \setminus \operatorname{rad}^{n+1}(X_0, X_n)$. The particular cases $n = 2, 3$ will be considered in more details.

Joint work with C. Chaio and S. Trepode (Universidad de Mar del Plata).

JOSÉ ANTONIO DE LA PEÑA, UNAM, Mexico
Coxeter transformations: from Lie algebras to singularity theory

Coxeter transformations play an important role in the theory of Lie algebras. Namely, the Weyl group is finite (resp. affine, contains a free subgroup) if the Coxeter elements are periodic (resp. have spectral radius 1, > 1). For a hereditary algebra $A = k\Delta$ associated to a quiver Δ without oriented cycles, the Coxeter transformation is induced from the Auslander-Reiten equivalence of the derived category $D^b(modA)$ to the Grothendieck group of A. The spectral properties of this transformation are essential to understand the representation theory of A. For canonical algebras A over the complex numbers, spectral properties of the Coxeter transformations are related to the classification of Fuchsian groups and their asociated singularities.

EDWARD GREEN, Virginia Tech
Noetherianity and Ext

The importance of the relationship between an algebra and its Ext-algebra is well established. On the other hand, little is known about which properties of the algebra or its representations imply, or are implied by the noetherianity of the Ext-algebra. The main thrust of this talk is the study of such properties. Particular attention is given to the case when the algebra is Koszul. Some of the results presented are given below.

We prove that if every module in $\operatorname{gr}(R)$ has a finitely generated Ext-module, $\bigoplus_{n \geq 0} \operatorname{Ext}_R^n(R/J, R/J)$, where J is the graded Jacobson radical of a standard graded algebra R, then R is left noetherian.

We prove that if R is a Koszul algebra of finite global dimension, then R being left noetherian is equivalent to every module $M = \bigoplus_i M_i$ in $\mathrm{gr}\, R$ has the property that for some n, the module $M_n \oplus M_{n+1} \oplus \cdots$ is linear.

MARK KLEINER, Syracuse University, Department of Mathematics, Syracuse, NY 13244-1150, USA
Reduced words in the Weyl group of a Kac–Moody algebra and preprojective representations of valued quivers

We discuss the interplay between the preprojective representations of a connected valued quiver, the $(+)$-admissible sequences of vertices, and the Weyl group. To each preprojective representation corresponds a canonical $(+)$-admissible sequence. A $(+)$-admissible sequence is the canonical sequence of some preprojective representation if and only if the product of simple reflections associated to the vertices of the sequence is a reduced word in the Weyl group. As a consequence, for any Coxeter element of the Weyl group associated to an indecomposable symmetrizable generalized Cartan matrix, the group is infinite if and only if the powers of the element are reduced words. The latter strengthens known results of Howlett and Fomin–Zelevinsky.

The talk is based on joint work with Helene R. Tyler and with Allen Pelley.

MARCELO LANZILOTTA, Unviersidad de la República, Iguá 1445, CP 11400, Montevideo, Uruguay
La technique de coupement à deux côtés pour la conjecture finitistique / Double cut approach for the finitistic conjecture

There is a famous conjecture ("Finitistic conjecture") in Representation theory of artin algebras:

"Given an artin algebra, its finitistic dimension is finite".

(The finitistic dimension is the supremum of the projective dimensions of the modules with finite projective dimension.)

This conjecture has more than 45 years, and is one of the most interesting problems at this area. In the talk we explain a new technique to treat the conjecture, using the Igusa–Todorov function. We exhibit a new family of quivers algebras with finite finitistic dimension.

AARON LAUVE, UQAM, LaCIM, C.P. 8888, Succ. Centre-Ville, Montréal, H3C 3P8
On novel ways to invert a matrix

Given an $n \times n$ matrix M over a (not necessarily commutative) field F and a candidate inverse M', the n^2 equations $M \cdot M' = I$, if solvable, define an inverse for M in $\mathrm{End}_F(F^n)$. For us, it is a small wonder that

(i) the solution is unique, and

(ii) the solution is the same as one would reach in solving the n^2 **different** equations $M' \cdot M = I$.

We are led to the following question: from the $2 \cdot n^2$ equations mentioned above, which choices of n^2 yield a unique solution M'? The case $n = 2$ is already interesting, involving a (reducible) Coxeter group of order eight, a nice lemma of Cohn's on the roots of noncommutative polynomials, ...

ROBERTO MARTINEZ-VILLA, Instituto de Matemáticas, UNAM, Morelia
Artin–Schelter regular algebras and categories

Having in mind non connected algebras, like the preprojective algebra, we introduce a generalization of the notion of a noncommutative regular algebra given by Artin and Schelter, we obtain some basic results and apply them to the the polynomial algebra. In order to include the category of finitely presented functors from the finitely generated modules over a finite

dimensional K-algebra, to the category of K-vector spaces, we extend the notion of Artin Schelter regular to additive categories. Finally, we give an application to the structure of the Auslander Reiten components.

The results presented here are part of a joint work with Oeyvind Solberg.

MARKUS SCHMIDMEIER, Florida Atlantic University
Nilpotent Linear Operators

In this talk I will present recent results from joint work with Claus Michael Ringel (Bielefeld) on nilpotent linear operators and their invariant subspaces.

Let k be a field. We consider triples (V, U, T) where V is a finite dimensional k-space, U a subspace of V and $T: V \to V$ a linear operator with $T^n = 0$ for some n, and such that $T(U) \subseteq U$. Thus, T is a nilpotent linear operator on V and U is an invariant subspace with respect to T.

If $v = \dim V$ and $u = \dim U$ then (v, u) is the dimension pair of the triple (V, U, T). It turns out that whenver the nilpotency index n is at most 6, then interesting properties about an indecomposable triple (V, U, T) can be read off from the dimension pair.

DAVID SMITH, Université de Sherbrooke, 2500 boul. de l'Université, Sherbrooke (Québec), J1K 2R1
Piecewise hereditary skew group algebras

The study of the representation theory of skew group algebras was started in the eighties with the works of de la Peña, and Reiten and Riedtmann. Given an algebra A and a group G acting on A, we define the skew group algebra $A[G]$. It turns out that $A[G]$ often retains many features from A, such as being representation-finite, being hereditary, being tilted or quasitilted, *etc*.

In this talk, we study the interplay between the skew group algebras and the so-called piecewise hereditary algebras, that is algebras A for which there exist a hereditary abelian category $\mathcal{H}$ and a triangle-equivalence between the derived categories of bounded complexes over A and $\mathcal{H}$. Those algebras, first studied by Happel, Rickard and Schofield and later by Happel, Reiten and Smalø, played a decisive role in the classification of selfinjective algebras of finite and tame representation type. We show that, under some assumptions, the skew group algebra $A[G]$ is piecewise hereditary when so is A.

The talk is based on joint work in progress with Julie Dionne and Marcelo Lanzilotta.

SONIA TREPODE, Universidad Nacional de Mar del Plata
On Auslander–Reiten Components with Bypasses

We study the Auslander–Reiten components which have a sectional bypass and we characterize them. We show that a bypass defines a new irreducible morphism.

Joint work with Claudia Chaio and Edson Ribeiro Alvares.

RITA ZUAZUA, UNAM Campus Morelia
When the graph subrings admit standard Noether normalizations?

In *Noether normalizations of some subrings of graphs* (Comm. Algebra **29**(2001), 5525–5534), Alcántar asked when a standard Noether normalizations of the monomial subring or edge subring of a graph exists. In this talk we will give an answer to the above question.

This is a joint work with Florian Luca.