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**VLADIMIR PESTOV**, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Ave., Ottawa, ON, K1N 6N5

*Urysohn metric space and Kirchberg approximation property*

The Urysohn universal metric space  $\mathbb{U}$  is a remarkable object, which can be described (Vershik) as the completion of the integers equipped with a random (or: generic) metric. In many regards, it is similar to the unit sphere  $\mathbb{S}^\infty$  of a separable Hilbert space  $\ell^2$ . There are however some properties long since established for the unit sphere (e.g. the distortion property) that remain open for the Urysohn space, and vice versa. We will discuss an example of the latter: Connes' Embedding Conjecture, whose analogue for the Urysohn space has been recently settled.

As a consequence of Kirchberg's work, Connes' Conjecture can be reformulated as follows: every pair of commuting subgroups of the unitary group  $U(\ell^2)$  can be approximated with pairs of commuting compact subgroups. In this form, the property (which we call Kirchberg property) makes sense for every topological group admitting a chain of compact subgroups with dense union. Even if such groups are very common among "infinite-dimensional" groups (the infinite symmetric group, the groups of measure and measure class preserving automorphisms, etc.), it seems the Kirchberg property has never been verified for *any* concrete example. In a recent joint work with V. V. Uspenskij, we have established the Kirchberg property for the group of isometries of the universal Urysohn metric space  $\mathbb{U}$ .