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The Lojasiewicz inequality for Higgs bundles

An important technique in the study of symplectic (and hyperkähler) quotients is to use the Morse theory of the norm-square of the moment map. The Lojasiewicz inequality is a key estimate in the process of proving that the gradient flow of a functional converges, which is essential in order for Morse theory to work. This technique was first used for certain infinite-dimensional problems by Leon Simon, and then extended by Johan Rade to study the gradient flow of the Yang–Mills functional in two and three dimensions. Here we extend Rade's version of the Lojasiewicz inequality to functionals which are invariant under a group action and also satisfy a certain ellipticity condition on the Hessian. In particular, this can be applied to the norm square of the hyperkähler moment map for Higgs bundles over a compact Riemann surface.