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Fractal Structure in Solitary Wave Interactions

The following scenario has been seen in many non-integrable, dispersive, nonlinear PDE over the last 25 years: two solitary waves are propagated on a collision course. Above some critical velocity v_c, they simply bounce off each other. Below v_c they may be captured and merge into a single localized mass, or they may interact a finite number of times before escaping each other's embrace. Whether they are captured, and how many times the solitary waves interact before escape, depends on the initial velocity in a complicated manner, often remarked, though never shown, to be a fractal (a chaotic scattering process). This has been observed in coupled NLS, sine-Gordon, phi^4, and others.

These PDE systems are commonly studied by (nonrigorously) deriving a reduced set of ODE that numerically reproduce this behavior. Using matched asymptotics and Melnikov integrals, we give asymptotic formulas for v_c and for certain salient features of the fractal structure. We derive a discrete-time iterated map through which the entire structure can be unravelled.

Joint with Richard Haberman, Southern Methodist University.