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Decay of waves on a warp product manifold with an unstable, closed geodesic surface

In this talk we compare the decay of solutions to the wave equation in R^3 to those of the wave equation on a three-dimensional warp product space, $(x,\omega) \in R \times S^2$ with metric $dx^2 + r(x)^2 d\omega^2$. If the radii of spheres, $r(x)$, has a unique minimum, then the sphere of minimal radius is a closed geodesic surface. Heuristically, this should impede the decay of waves, since waves follow the paths of geodesics. Indeed, Ralston has shown that initial data can be chosen for which an arbitrarily large percentage of the energy (H^1 density) remains within a neighbourhood of the geodesic for arbitrary long periods of time. Thus, if decay estimates hold, there must be some loss of regularity, with higher derivatives used to control the decay of localised energy. The conformal charge is a weighted H^1 norm defined by analogy to R^n, and its growth is controlled by the time integral of the energy near the geodesic surface. Using refinements of previous local decay arguments, the conformal charge can be shown to be bounded with an epsilon loss of regularity if the geodesic surface is unstable. This gives the same rate of decay for certain L^p norms as in R^n. This argument uses vector field techniques and can be extended to small data, nonlinear problems under additional assumptions on the growth of $r(x)$. Since vector field techniques are analogous to commutator methods, we expect that similar methods will apply to the NLS on manifolds.