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Old and New Morrey Spaces with Heat Kernel Bounds

In this talk I will address my work joint with D. Xuong and L. Yan. More precisely, given $p \in [1, \infty)$ and $\lambda \in (0, n)$, we discuss Morrey space $L^{p,\lambda}(\mathbb{R}^n)$ of all locally integrable complex-valued functions f on \mathbb{R}^n such that for every open Euclidean ball $B \subset \mathbb{R}^n$ with radius r_B there are numbers $C = C(f)$ (depending on f) and $c = c(f, B)$ (relying upon f and B) satisfying

$$r_B^{-\lambda} \int_B |f(x) - c|^p dx \leq C$$

and derive old and new, two essentially different cases arising from either choosing $c = f_B = |B|^{-1} \int_B f(y) dy$ or replacing c by $P_{t_B}(x) = \int_{t_B} p_{t_B}(x, y) f(y) dy$ —where t_B is scaled to r_B and $p_t(\cdot, \cdot)$ is the kernel of the infinitesimal generator L (taking the Schroedinger operator as a special one) of an analytic semigroup $\{e^{-tL}\}_{t \geq 0}$ on $L^2(\mathbb{R}^n)$. Consequently, we are led to simultaneously characterize the old and new Morrey spaces, but also to show that for a suitable operator L , the new Morrey space is equivalent to the old one.