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On the Small Ball Problem

We consider Haar functions in the unit cube in three dimensions, normalized in L^∞. The question at hand is a 'non-trivial' lower bound on the L^∞ norm of the sum

$$\sum_{|R|=2^{-n}} a_R h_R(x).$$

The key point of the sum is that is formed over rectangles of a fixed volume—this is the 'Hyperbolic' assumption. We prove that for some $\eta > 0$, we have the estimate

$$\Big\| \sum_{|R|=2^{-n}} a_R h_R(x) \Big\|_\infty > c n^{-1+\eta} 2^{-n} \sum_{|R|=2^{-n}} |a_R|$$

($\eta = 0$ is the 'trivial' estimate). In a prior result of J. Beck, a famous and famously difficult result, established a logarithmic gain over the trivial estimate. We simplify and extend Beck's argument to prove this result.

Joint work with Dmitry Bilyk.