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Partial hyperbolicity and ergodicity in dimension three

In [?] we proved the Pugh–Shub conjecture for conservative partially hyperbolic diffeomorphisms with one-dimensional center. That is, stably ergodic diffeomorphisms are dense among the conservative partially hyperbolic ones. Can we describe this abundance of ergodicity more accurately?

More precisely:

Problem *Which 3-dimensional manifolds support a non-ergodic partially hyperbolic diffeomorphism?*

We conjecture that the answer to this question is that the only such manifolds are the mapping tori of diffeomorphisms commuting with an Anosov one. In the other cases, being partially hyperbolic would automatically imply ergodicity. We prove this for a family of manifolds:

Theorem *Let $f: N \to N$ be a conservative partially hyperbolic C^2 diffeomorphism where $N \neq \mathbb{T}^3$ is a compact 3-dimensional nilmanifold. Then, f is ergodic.*

Sacksteder [?] proved that certain affine diffeomorphisms of nilmanifolds are ergodic. These examples are partially hyperbolic.

Some of our results apply to other manifolds and we obtain, for instance, that every conservative partially hyperbolic diffeomorphism of $\mathbb{S}^3$ is ergodic but this is probably a theorem about the empty set.

This is a joint work with María Alejandra Rodriguez Hertz and Raúl Ures.

References

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