MARIA G. REZNIKOFF, School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, Georgia 30332

Slow Motion of Gradient Flows

Sometimes physical systems exhibit "metastability," in the sense that states get drawn toward so-called metastable states and are trapped near them for a very long time. A familiar example is the one-dimensional Allen–Cahn equation: Initial data is drawn quickly to a "multi-kink" state and the subsequent evolution is exponentially slow. The slow coarsening has been analyzed by Carr & Pego, Fusco & Hale, Bronsard & Kohn, and X. Chen.

In general, what causes metastability? Our main idea is to convert information about the energy landscape (statics) into information about the coarsening rate (dynamics). We give sufficient conditions for a gradient flow system to exhibit metastability. We then apply this abstract framework to give a new analysis of the 1-d Allen–Cahn equation. The central ingredient is to establish a certain nonlinear energy–energy–dissipation relationship. One benefit of the method is that it gives a natural proof of the fact that exponential closeness to the multi-kink state is not only propagated, but also generated.

This work is joint with Felix Otto, University of Bonn.