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Integrality gaps of SDP for Vertex Cover and relations to ℓ_1 embeddability of negative type metrics

We study various SDP formulations for vertex cover by adding different constraints to the standard formulation. We show that vertex cover cannot be approximated better than $2 - o(1)$ even when we add the so-called pentagonal inequality constraints to the standard SDP formulation, en route answering an open question of Karakostas. We further show the surprising fact that by strengthening the SDP with the (intractable) requirement that the metric interpretation of the solution is an ℓ_1 metric, we get an exact relaxation (integrality gap is 1), and on the other hand if the solution is arbitrarily close to being ℓ_1 embeddable, the integrality gap may be as big as $2 - o(1)$. Finally, inspired by the above findings, we use ideas from the integrality gap construction of Charikar to provide a family of simple examples for negative type metrics that cannot be embedded into ℓ_1 with distortion better than $8/7 - \epsilon$. To this end we prove a new isoperimetric inequality for the hypercube.