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A simply connected counter-example to Ganea's Conjecture of Smallest Dimension

For a topological space X, $\mathrm{cat}(X)$ is one less than the least number of open sets in X that it takes to cover X. Ganea conjectured that for any X, $\mathrm{cat}(X \times S^n) = \mathrm{cat}(X) + 1$. In 1997 Iwase constructed a series of counterexamples using the instability of certain Hopf invariants. The least dimension of the counterexamples was ten. On the other hand results of Vandembrouq imply that no simply connected counterexamples exist with dimension less that or equal to five. We construct a counter-example of dimension seven. Instead of unstable Hopf invariants, the example comes from a Hopf invariant stably factoring through another map. We will also discuss the remaining question of counterexamples of dimension six.

This is joint work with Hugo Rodriguez.