
Probability
Probabilité
(Org: **Ana Meda** (UNAM) and/et **Ed Perkins** (UBC))

MARTIN BARLOW, University of British Columbia, Vancouver, BC. Canada

Invariance principles for the random conductance model

The Random Conductance Model (RCM) is a model of a reversible or symmetric random walk in a random environment. i.i.d. weights μ_e are assigned to the edges in \mathbb{Z}^d . A random walk X is then run, which makes its jumps with probabilities proportional to the edge weights.

This model is now quite well understood in the special cases when either the law of μ_e is concentrated on $[0, 1]$ or $1, \infty$. I will discuss what happens in these cases, and in particular in the case when $E(\mu_e) = \infty$.

BEGOÑIA FERNÁNDEZ, Universidad Nacional Autónoma de México

Estimates for the Probability Itô Processes Remain Around a Curve, and Applications to Finance

We consider a Brownian Motion $W = (W^i)_{i \in N}$ and an adapted process of dimension n $(X_t)_{t \geq 0}$. We define $\tau_R = \inf\{t : |X_t - x_t| \geq R_t\}$, where $x_t, t \geq 0$ is a deterministic differentiable curve in R^n and $R_t > 0, t \geq 0$ a ratio that depends on time. Assume that until τ_R the process X is a solution of the equation

$$X_{t \wedge \tau_R} = x + \sum_{j=1}^{\infty} \int_0^{t \wedge \tau_R} \sigma_j(s, \omega, X_s) dW_s^j + \int_0^{t \wedge \tau_R} b(s, \omega, X_s) ds,$$

where the coefficients σ_j and b are adapted, locally bounded and $(t, x) \rightarrow \sigma_j(t, \omega, x)$ are Lipschitz continuous and satisfy

$$\gamma_t \geq \sigma \sigma^*(t \wedge \tau_R, \omega, X_{t \wedge \tau_R}) \geq \lambda t.$$

We obtain lower bounds of the form:

$$\exp \left[-Q_n^1 \left(1 + \int_0^{T+r} F_x^1(t) dt \right) \right] \leq P(\tau_R > T) \quad (1)$$

where Q_n^1 is a constant and F_x^1 is a function that depends on x_t and R_t .

We apply the results to obtain lower bounds for option prices for stochastic volatility models.

Joint work with V. Bally and A. Meda.

JORGE GARCIA, California State University Channel Islands

Uniform Exponential Tightness Vs. Exponential Stochastic Boundedness

In this talk some of the main parallels between weak convergence and large deviations will be explained, particularly, weak convergence of stochastic integrals (in the sense of Kurtz–Protter) and large deviations for sequences of stochastic integrals. Conditions will be given under which a sequence of stochastic processes (X_n, Y_n) implies large deviations for $\int X_n dY_n$.

JULIO CÉSAR GARCÍA-CORTE, Universidad Autónoma Metropolitana, Iztapalapa

Invariant States of the Asymmetric Exclusion Quantum Dynamical Semigroup

In this talk we present the qualitative properties of the Asymmetric Exclusion Quantum Dynamical Semigroup: its qualitative properties, their invariant states, convergence to the equilibrium, and characterization of the domain attractions of each of the invariant states.

This is a joint work with Roberto Quezada Batalla and Lepoldo Pantaleon Martinez.

JOSÉ MARÍA GONZÁLEZ-BARRIOS, Universidad Nacional Autonoma de Mexico, IIMAS, Circuito Escolar s/n, Mexico D.F. 04510, Mexico

Associativity and Symmetry of Copulas

In this talk we will introduce two new statistics A_π^n and T_n defined for random samples of size n , of a pair of continuous random variable (X, Y) with copula C . The statistics measure the associativity and symmetry of the samples respectively, that is, if the copula satisfies

$$C(x, C(y, z)) = C(C(x, y), z) \quad \text{for every } x, y, z \in [0, 1]$$

and

$$C(x, y) = C(y, x) \quad \text{for every } x, y \in [0, 1].$$

These conditions are necessary for the copula C to belong to the Archimedean family.

We will study the properties of the new statistics, and we will include some applications.

DANIEL HERNÁNDEZ-HERNÁNDEZ, CIMAT

Robust utility maximization in a financial market with prices driven by Lévy processes: A dual approach

We deal with the dynamic maximization of a robust utility function which penalize the possible probabilistic models. The context will be of a market model with prices determined by an external factor which is driven by a Lévy stochastic integral. We characterize first the classes of measures (densities) related to such a market. Once it is established the relation of the penalty associated to the utility function with a convex risk measure, we are able to use duality theory recently developed for an optimal investment in an risk and ambiguity averse setting.

ONÉSIMO HERNÁNDEZ-LERMA, Mathematics Department, CINVESTAV-IPN, A. Postal 14-740, Mexico, D.F. 07000, Mexico

Stochastic Control Systems with Long-Run Average Criteria

Stochastic control problems with long-run average criteria (also known as *ergodic criteria*) were introduced by Richard Bellman (1957) in the context of a manufacturing process, and nowadays play a predominant role in control applications to queueing systems, telecommunication networks, and economic and financial problems, to name a few.

This talk presents some recent advances on *discrete* and *continuous* time stochastic control systems with long-run average criteria, including overtaking (or catching-up) optimality, bias optimality, discount-sensitive criteria, and the existence of average optimal strategies with minimum variance.

ALEXANDER HOLROYD, UBC and Microsoft

Finitary Colouring

Suppose that the vertices of Z^d are assigned random colours via a finitary factor of i.i.d. random vertex-labels. That is, the colour of vertex v is determined by a rule which examines the labels within a finite (but random and perhaps unbounded) distance R of v , and the same rule applies at all vertices. We investigate the tail behaviour of R if the coloring is required to

be proper (that is, adjacent vertices receive different colours). Depending on the dimension and the number of colours, the optimal tail is either power law or super-exponential.

ROBERT MASSON, UBC, 1984 Mathematics Road, Vancouver, BC, V6T 1Z2

Second moment estimates for the growth exponent of planar loop-erased random walk

The loop-erased random walk \widehat{S}^n is the process obtained by running a random walk in Z^d from the origin to the first exit time of the ball of radius n and then chronologically erasing its loops. If we let M_n denote the number of steps of \widehat{S}^n then the growth exponent α is defined to be such that $E[M_n]$ grows like n^α . The value of α depends on the dimension d . In this talk we'll focus on $d = 2$ where it's been shown that $\alpha = 5/4$. We will establish a second moment result and use it to get estimates for the probability that M_n is close to its mean. Namely, we show that there exists $0 < p < 1$ such that for all n and λ large, $P(M_n < \lambda^{-1}E[M_n]) < p^{\lambda^{1/6}}$.

This is joint work with Martin Barlow.

JEREMY QUASTEL, University of Toronto

Scaling exponent of KPZ

We obtain bounds at the expected order for the variance of the logarithm of the solution of the stochastic heat equation and correlation functions of its derivative, which are understood as the solutions of the Kardar–Parisi–Zhang and Stochastic Burgers equations.

Joint work with Marton Balazs and Timo Seppalainen.

DENIZ SEZER, University of Calgary, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4, Canada

Conditioning Super-Brownian Motion on its boundary statistics and a class of "weakly" extreme X -harmonic functions

Let X be a super-Brownian motion (SBM) defined on \mathbb{R}^n and (X_D) be its exit measures indexed by sub-domains of \mathbb{R}^d . We pick a bounded sub-domain D , and condition the super-brownian motion inside this domain on its "boundary statistics", random variables defined on an auxiliary probability space generated by sampling from the exit measure X_D . Among these, two particular examples are conditioning on a Poisson random measure with intensity βX_D , and X_D itself. We find the conditional laws as h-transforms of the original SBM law using X -harmonic functions.

The X -harmonic function H^ν corresponding to conditioning on $X_D = \nu$ is of special interest, as it can be thought as the analogue of the Poisson kernel. An open problem is to show that H^ν is extreme at least for some ν when D is a smooth domain. An equivalent problem is to show that the tail sigma field of SBM in D is trivial with respect to P^ν . We prove a weaker version of this result using an approximation, first by conditioning on a Poisson random measure with intensity nX_D and then letting n go to infinity. We show that for any A in the tail sigma field of X , $P^{X_D}(A) = 0$ or 1 almost surely.

CHRISTINE SOTEROS, University of Saskatchewan, 106 Wiggins Road, Saskatoon, SK, S7N 5E6

Knotting Probability for Stretched Polygons in a Lattice Tube

The topological entanglements of polygons confined to a lattice tube and under the influence of an external tensile force f will be examined. The tube constraint allows us to prove a pattern theorem via transfer matrix arguments for any arbitrary fixed value of f . The resulting stretched polygon pattern theorem can then be used to show that the knotting probability of an n -edge stretched polygon confined to a tube goes to one exponentially as $n \rightarrow \infty$. Thus as $n \rightarrow \infty$ when polygons are influenced by a force f , no matter its strength or direction, topological entanglements, as defined by knotting, occur with high probability.

This is joint work with M. Atapour and S. G. Whittington.

JOHN WALSH, UBC

The Roughness and Smoothness of Numerical Solutions to the Stochastic Heat Equation

The stochastic heat equation is the heat equation driven by white noise. We consider its numerical solutions using the finite difference method. Its true solutions are Hölder continuous with parameter $(\frac{1}{2} - \epsilon)$ in the space variable, and $(\frac{1}{4} - \epsilon)$ in the time variable. We show that the numerical solutions share this property in the sense that they have non-trivial limiting quadratic variation in x and quartic variation in t . These variations are discontinuous functionals on the space of continuous functions, so it is not automatic that the limiting values exist, and not surprising that they depend on the exact numerical schemes that are used; it requires a very careful choice of scheme to get the correct limiting values. In particular, part of the folklore of the subject says that a numerical scheme with excessively long time-steps makes the solution much smoother. We make this precise by showing exactly how the length of the time-steps affects the quadratic and quartic variations.

This is joint work with Yuxiang Chong.