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The Roughness and Smoothness of Numerical Solutions to the Stochastic Heat Equation

The stochastic heat equation is the heat equation driven by white noise. We consider its numerical solutions using the finite difference method. Its true solutions are Hölder continuous with parameter $(\frac{1}{2} - \epsilon)$ in the space variable, and $(\frac{1}{4} - \epsilon)$ in the time variable. We show that the numerical solutions share this property in the sense that they have non-trivial limiting quadratic variation in x and quartic variation in t. These variations are discontinuous functionals on the space of continuous functions, so it is not automatic that the limiting values exist, and not surprising that they depend on the exact numerical schemes that are used; it requires a very careful choice of scheme to get the correct limiting values. In particular, part of the folklore of the subject says that a numerical scheme with excessively long time-steps makes the solution much smoother. We make this precise by showing exactly how the length of the time-steps affects the quadratic and quartic variations.

This is joint work with Yuxiang Chong.