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*Conditioning Super-Brownian Motion on its boundary statistics and a class of "weakly" extreme  $X$ -harmonic functions*

Let  $X$  be a super-Brownian motion (SBM) defined on  $\mathbb{R}^n$  and  $(X_D)$  be its exit measures indexed by sub-domains of  $\mathbb{R}^d$ . We pick a bounded sub-domain  $D$ , and condition the super-brownian motion inside this domain on its "boundary statistics", random variables defined on an auxiliary probability space generated by sampling from the exit measure  $X_D$ . Among these, two particular examples are conditioning on a Poisson random measure with intensity  $\beta X_D$ , and  $X_D$  itself. We find the conditional laws as h-transforms of the original SBM law using  $X$ -harmonic functions.

The  $X$ -harmonic function  $H^\nu$  corresponding to conditioning on  $X_D = \nu$  is of special interest, as it can be thought as the analogue of the Poisson kernel. An open problem is to show that  $H^\nu$  is extreme at least for some  $\nu$  when  $D$  is a smooth domain. An equivalent problem is to show that the tail sigma field of SBM in  $D$  is trivial with respect to  $P^\nu$ . We prove a weaker version of this result using an approximation, first by conditioning on a Poisson random measure with intensity  $nX_D$  and then letting  $n$  go to infinity. We show that for any  $A$  in the tail sigma field of  $X$ ,  $P^{X_D}(A) = 0$  or  $1$  almost surely.