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Computation of Hermite–Pade interpolants by iterated polynomial interpolation

Let I be an open interval and $f\colon I \to \mathbb{R}$ a sufficiently differentiable function such that $f(x) \neq 0$ for $x \in I$. Suppose that $u_0, u_1, u_2, \ldots$ is a sequence of monic polynomials such that for $k \geq 0$ we have that u_k divides u_{k+1} and has all its roots in I.

If g is a sufficiently differentiable function defined on I, we denote by $H(g, u_k)$ the polynomial of smallest degree that interpolates g at the multiset of roots of u_k in the sense of Hermite.

We construct a sequence of polynomials $p_0, p_1, p_2, \ldots$ defined as follows:

$$\begin{gathered} p_0 = H(f, u_0), \quad p_1 = H(p_0/f, u_1), \\ \text{for } k \text{ even} \quad p_k = H(f\, p_{k-1}, u_k), \text{ and} \\ \text{for } k \text{ odd} \quad p_k = H(p_{k-1}/f, u_k). \end{gathered}$$

We show that for each even integer k the rational function p_k/p_{k+1} is a Hermite–Padé interpolant for f at the multiset of roots of u_k. We present some examples and numerical results. We also consider some particular choices for the sequence u_k and indicate some possible modifications of our construction.