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The Bogdanov–Takens bifurcation in a class of traffic flow models

Macroscopic traffic models are based in the analogy with a continuous 1-dimensional flow. Conservation of number of cars leads to conservation of mass and the Navier–Stokes equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial x} = 0 \tag{1}$$

$$\rho\Big(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\Big) = \frac{\partial}{\partial x}\Big(\eta\frac{\partial V}{\partial x}\Big) - \frac{\partial p}{\partial x} + X \tag{2}$$

where $\rho(t,x)$ is density, $V(t,x)$ the average velocity of cars, η the "viscosity" and p the local pressure, being proportional to the variance ("temperature") of the traffic $\Theta(x,t)$ and to the gradient velocity; in the simplest case $p = \theta_0\rho - \eta_0\frac{\partial V}{\partial x}$ with constants Θ_0, η_0. The "external forces" in the Kerner–Konhauser are represented by driver's tendency to acquire a safe velocity $V_e(\rho)$ with a relaxation time τ, $X = \rho\frac{V_e(\rho)-V}{\tau}$, where $V_e(\rho)$ is the empirical "fundamental relationship" usually a monotone decreasing function bounded from below.

By performing the change of variables $z = x+v_g t$, solutions of (1) in the form of travelling waves are reduced to: $\rho(v+v_g) = q_g$ with parameters v_g, q_g, and solutions of (**??**) are reduced to a dynamical system, written in adimensional variables as

$$\begin{aligned}\frac{dv}{dz} &= y\\ \frac{dy}{dz} &= \lambda q_g\Big[1 - \frac{\theta_0}{(v+v_g)^2}\Big]y - \mu q_g\Big(\frac{v_e(v)-v}{v+v_g}\Big).\end{aligned} \tag{3}$$

Here $v_e(v)$ is de adimensional version of $V_e(\rho)$.

We prove that there exist a curve in the parameter space v_g–q_g–Θ_0 such that system (3) undergoes a Bagdanov–Takens bifurcation. In particular we prove the existence of Hopf and homoclinic bifurcations.

This is a joint work with Patricia Saavedra and Rosa María Velasco (UAM–I).