PACO LARRIÓN, UNAM, Ciudad Universitaria, México, D.F. Mexico
The fundamental group of the clique graph

The *cliques* of a graph G are its maximal complete subraphs or, rather, their vertex sets. The *clique graph* of G is the intersection graph $K(G)$ of its cliques, so the vertices of $K(G)$ are the cliques of G, and two of them are neighbours in $K(G)$ if they are distinct and share at least one vertex. The *iterated clique graphs* of G are recursively defined by $K^0(G) = G$ and $K^{n+1}(G) = K\bigl(K^n(G)\bigr)$.

We are interested in the dynamical behaviour of a graph G under the clique graph operator K. For instance, G is K*-null* if some iterated clique graph $K^n(G)$ is the trivial graph K_1. More generally, G is K*-convergent* if $K^m(G) \cong K^n(G)$ for some pair $m < n$. It is easy to see that G is not K-convergent precisely when it is K*-divergent*, in the sense that the order of $K^n(G)$ tends to infinity with n.

The complete subgraphs of G, viewed as vertex sets, form a simplicial complex. Via the geometric realization of this complex, we can consider the graph G as a topological space. Erich Prisner proved in 1992 that the first modulo two homology group of $K(G)$ is the same as that of G, and we proved recently the stronger statement that the fundamental group of $K(G)$ coincides with that of G. This gives a necessary condition for a graph to be K-null.

The talk is about joint work with M. A. Pizaña and R. Villarroel-Flores.