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Periodic and Bloch solutions to a magnetic nonlinear Schrödinger equation

We consider the equation

$$(\wp_A) \quad (-i\nabla + A)^2 u + Vu = |u|^{p-2}u,$$

where $A \in C^{1,\alpha}(\mathbb{R}^N, \mathbb{R}^N)$ is the magnetic potential associated to a magnetic field $B = \mathrm{curl}(A)$ and $V \in C^{0,\alpha}(\mathbb{R}^N)$ is an electric potential. We assume that A and V are 2π-periodic in each variable, $V > 0$, and $p \in (2, 2^*)$ with $2^* := \infty$ if $N = 2$, $2^* := \frac{2N}{N-2}$ if $N \geq 3$. We shall address two questions: the gauge-dependence problem for 2π-periodic solutions $u\colon \mathbb{R}^N \to \mathbb{C}$, and the multiplicity of Bloch solutions. Unlike the nonperiodic case where problem $(\wp_A)$ is basically independent of A (it is gauge invariant), in the periodic case this is far from being true. Under some assumptions on A we show that, if there exists a one-to-one correspondence between the 2π-periodic solutions of $(\wp_A)$ and those of $(\wp_{A+z})$ preserving their absolute value, then z lies in a subset of measure zero of $\mathbb{R}^N$. We use this fact to show the existence of an uncountable set of Bloch solutions with real quasimomentum having small energy.

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