John and Nirenberg (1961) introduced the space BMO(Q) and a larger space which we call the John–Nirenberg space with exponent p and denote by $JN_p(Q)$, where Q is a finite cube in \mathbb{R}^n . They proved two lemmas for functions in BMO(Q) and $JN_p(Q)$ respectively. The first one characterizes functions in BMO(Q) in terms of the exponential decay of the distribution function of their oscillations. The second shows that any function in $JN_p(Q)$ is in weak $L^p(Q)$.

We first give a new proof for John–Nirenberg lemma II on \mathbb{R}^n by using a dyadic maximal operator and a good lambda inequality. Then, we discuss the space JN_p and the corresponding lemma in the context of a doubling metric measure space.

Joint work with D. Aalto, L. Berkovits, O. E. Maasalo.

HONG YUE, Trine University, 1 University Ave., Angola, IN, 46703, USA *A L^p*-version of the John–Nirenberg Lemma in Metric Spaces