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Refinements of Milnor's Fibration Theorem for complex singularities

Given a holomorphic map-germ $f\colon (\mathbb{C}^n, \underline{0}) \to (\mathbb{C}, 0)$ with a critical value at $0 \in \mathbb{C}$, there are two equivalent ways of defining its Milnor fibration. The first is:

$$\phi = \frac{f}{|f|}\colon \mathbb{S}_\epsilon \setminus K \longrightarrow \mathbb{S}^1, \tag{1}$$

where $K = f^{-1}(0) \cap \mathbb{S}_\epsilon$ is the link. The other, given essentially by Milnor himself by:

$$f\colon N(\epsilon, \eta) \longrightarrow \partial\mathbb{D}_\eta, \tag{2}$$

where $\epsilon \gg \eta > 0$ are sufficiently small, $\mathbb{D}_\eta \subset \mathbb{C}$ is the disc of radius η around $0 \in \mathbb{C}$, $\mathbb{B}_\epsilon$ is the ball of radius ϵ around $\underline{0} \in \mathbb{C}^n$ and $N(\epsilon, \eta)$ is the *Milnor tube* $\mathbb{B}_\epsilon \cap f^{-1}(\partial\mathbb{D}_\eta)$. (We remark that Milnor only proved that the fibres of (2) are equivalent to those of (1), and not that (2) is actually a fibre bundle; this was certainly known to Milnor when f has an isolated critical point, and later completed by Lê.)

We show that there is a canonical decomposition of the whole ball $\mathbb{B}_\epsilon$ into real analytic hypersurfaces X_θ that spin around their "axis" $V_\epsilon = f^{-1}(0) \cap \mathbb{B}_\epsilon$ forming a kind of "open-book" with singular binding. Using this decomposition we improve, or refine, Milnor's fibration theorem in several directions.

This is joint work with J. Seade and J. Snoussi.