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Hyperbolicity and exponential convergence of the Lax Oleinik semigroup

Consider a convex superlinear Lagrangian $L\colon TM \to \mathbb{R}$ on a compact manifold M. It has been shown that there is a unique number c such that the Lax Oleinik semigroup $\mathcal{L}_t\colon C(M,\mathbb{R}) \to C(M,\mathbb{R})$ defined by

$$\mathcal{L}_t v(x) = \inf\left\{ v\big(\gamma(0)\big) + \int_0^t L(\gamma,\dot\gamma) + ct : \gamma\colon [0,t] \to M \text{ is piecewise } C^1, \gamma(t) = x \right\}$$

has a fixed point. Moreover for any $v \in C(M,\mathbb{R})$ the uniform limit $\tilde v = \lim_{t\to\infty} \mathcal{L}_t v$ exists.

Theorem 1 *Assume that the Aubry set consists in a finite number of hyperbolic periodic orbits or critical points of the Euler–Lagrange flow. Then, there is $\mu > 0$ such that for any $v \in C(M,\mathbb{R})$ there is $K > 0$ such that*

$$\|\mathcal{L}_t v - \tilde v\|_u \le K e^{-\mu t} \quad \forall t \ge 0.$$

We believe the reciprocal holds but for the moment we only have the proof for a mechanical Lagrangian.

Theorem 2 *Let $L\colon TM \to \mathbb{R}$ given by $L(x,v) = \frac{1}{2}v^2 - V(x)$ with*

$$\max_x V(x) = c, \quad V^{-1}(c) = \{x_1, \dots, x_m\}.$$

Suppose that there is $\mu > 0$ such that for any $v \in C(M,\mathbb{R})$ there is $K > 0$ such that

$$\|\mathcal{L}_t v - \tilde v\|_u \le K e^{-\mu t} \quad \forall t \ge 0.$$

Then $(x_i, 0)$, $i = 1, \dots m$ is a hyperbolic critical point of the Euler–Lagrange flow.