Mathematical Physics

(Org: **D. Brydges** (UBC), **S. Sontz** (CIMAT) and **Carlos Villegas** (UNAM-Cuernavaca))

DAVID BRYDGES, University of British Columbia
A combinatorial generalisation of Cramer's rule

We review a result of G. X. Viennot, Lecture Notes in Mathematics **1234**(1986), and comment on its significance for statistical mechanics: a ratio of generating functions for disjoint oriented loops in a finite graph can be expressed in terms of the generating function of a single path in the graph weighted according to loops in the path. The result is a generalisation of Cramer's formula for the inverse of a matrix.

JAIME CRUZ SAMPEDRO, Universidad Autónoma del Estado de Hidalgo
Embedded Eigenvalues of Continuous and Discrete Schrödinger Operators

First we present general results about the instability of embedded eigenvalues in the continuum, and then we treat in more detail the instability of embedded eigenvalues of one dimensional Schroedinger operators, with Wigner–von Neumann-like potentials, both in the continuous and the discrete cases.

RAFAEL DEL RIO, UNAM
Sturm–Liouville operators in the half axis with local perturbations

We give conditions which imply equivalence of the Lebesgue measure with respect to a measure μ generated as an average of spectral measures corresponding to Sturm–Liouville operators in the half axis. We apply this to prove that some spectral properties of these operators hold for large sets of boundary conditions if and only if they hold for large sets of positive local perturbations.

This is joint work with O. Tchebotareva.

RICHARD FROESE, University of British Columbia
AC spectrum for Schrödinger operators on tree-like graphs

I will discuss recent proofs, obtained with Hasler and Spitzer, of the existence of absolutely continuous spectrum for Schrödinger operators on graphs.

ANTONIO HERNANDEZ, IIMAS–UNAM, Apdo. Postal 20-726, Mexico City 01000, Mexico
Symmetry breaking and adiabatic invariants

We will discuss adiabatic momentum maps in the context of examples of mechanical systems with approximate symmetry. A procedure for averaging the "variational principle" will be described.

DIMITRI JAKOBSEN, McGill, Dept. of Math, 805 Sherbrooke W, Montreal, QC, H3A 2K6, Canada
Estimates from below for spectral function and error term in Weyl law

We obtain asymptotic lower bounds for the spectral function of the Laplacian on compact manifolds. In the negatively curved case, thermodynamic formalism is applied to improve the estimates. Our results can be considered pointwise versions (on a general manifold) of Hardy's lower bounds for the error term in the Gauss circle problem. We next obtain a lower bound for the remainder in Weyl's law on negatively curved surfaces. On higher-dimensional negatively curved manifolds, we prove a similar bound for the oscillatory error term. Our approach uses wave trace asymptotics, equidistribution of closed geodesics and small-scale microlocalization.

ROBERT MOODY, University of Victoria, Victoria, BC
Between order and disorder: the mathematics of quasicrystals

Quasicrystals are materials that lie somewhere between crystals and disordered materials. This talk is an introduction to the mathematics that has been created to model and explain them. We will start with various characterizations of mathematical crystals, where the underlying structure is based on lattices, and show how nicely these can be generalized to encompass some of this intermediate world of aperiodic order.

A characteristic feature of quasicrystals is their crystal-like diffraction, which is deeply related to their internal order. We will indicate some of the known ways of producing aperiodic pure point diffractive sets and the current state of trying to characterize them.

At the end of the day, aperiodic order seems to be a reconciliation of precise local order and average global order. We will show how, through the use of dynamical systems, this idea can be made more precise.

JEREMY QUASTEL, Dept. of Maths., University of Toronto, 40 St. George, Toronto
Effect of Noise on Traveling Fronts in the Fisher–KPP equation

KPP-type reaction-diffusion equations perturbed by noise have random traveling fronts. We compute the speed asymptotically for small values of the noise. As conjectured by Brunet and Derrida, the slowdown is as the inverse square of the logarithm of the noise, with an explicit constant.

LUIS SILVA, IIMAS–UNAM Apartado Postal 20-726 Mexico DF 01000
Applications of M. G. Krein's Theory of Entire Operators to Sampling Theory

The Whittaker–Shannon–Kotel'nikov Sampling Theorem gives a formula for reconstructing Paley–Wiener functions from their values at a discrete set of points (samples). This theorem has been extended and generalized in various ways.

In this talk we consider a generalization of the Whittaker–Shannon–Kotel'nikov Sampling Theorem on the basis of a particular class of simple symmetric operators with deficiency indices $(1, 1)$. The theory of this class of operators is due to M. G. Krein.

This is a joint work with Julio H. Toloza.

GORDON SLADE, University of British Columbia, Vancouver
Invasion percolation on a tree

Invasion percolation is a stochastic growth process which produces a random infinite subgraph—the invaded region—of a given infinite graph G. We consider the case where G is a regular tree and study the large-scale properties of the invaded region. Viewed far from the origin, the invaded region looks locally like a large critical percolation cluster. But surprisingly, we prove that the global structure of the invaded region is dramatically different than that of the incipient infinite percolation cluster.

This is joint work with O. Angel, J. Goodman and F. den Hollander.

STEPHEN SONTZ, CIMAT, Guanajuato, Gto., Mexico
Heat kernel analysis in a deformation of quantum mechanics

We present a μ-deformation of quantum mechanics based on Dunkl operators, as studied by Rosenblum. This includes a μ-deformed Segal–Bargmann space and an asssociated μ-deformed Segal–Bargmann transform. We show the relation of these structures to heat kernel analysis, following ideas introduced by Hall.

JULIO TOLOZA, Universidad Nacional Autónoma de México, Ciudad Universitaria, México D.F.
Absence of continuous spectrum on a class of unbounded Jacobi operators

We establish sufficient conditions for self-adjointness on a class of unbounded Jacobi operators defined by matrices with main diagonal sequence of very slow growth and rapidly growing off-diagonal entries. With some additional assumptions, we also prove that these operators have only discrete spectrum.

ALEXANDER TURBINER, Nuclear Science Institute, UNAM
Anharmonic oscillator and double-well potential: approximating eigenfunctions

A simple uniform approximation of the logarithmic derivative of the ground state eigenfunction for both the quantum-mechanical anharmonic oscillator and the double-well potential given by $V = m^2x^2 + gx^4$ at arbitrary $g \geq 0$ for $m^2 > 0$ and $m^2 < 0$, respectively, is presented. It is shown that if this approximation is taken as unperturbed problem it leads to an extremely fast convergent perturbation theory. A connection with WKB approximation is briefly discussed.

Dedicated to the memory of Professor Felix A. Berezin.

CARLOS VILLEGAS, UNAM, Matemáticas, Cuernavaca
Asymtotics of clusters of eigenvalues for perturbations of the hydrogen atom Hamiltonian

We present in this talk a limiting eigenvalue distribution theorem for the Schrödinger operator of the hydrogen atom (with the Planck parameter $\hbar$ included) plus ϵ times a bounded continuous function Q. By considering suitable dilation operators, we prove that taking $\epsilon = O(\hbar^2)$ we obtain well defined clusters of eigenvalues around the energy $E = -1/2$ whose limiting distribution involves the Radon transform of the function Q along the classical orbits of the Kepler problem with energy $E = -1/2$ with respect to an integration over the space of geodesics of the 3-sphere S^3. The idea of the proof involves a well known unitary transformation from the Hilbert space generated by the bound states of the hydrogen atom onto $L^2(S^3)$ and coherent states on the sphere S^3. We will comment on the generalization of the theorem above to the n-dimensional case and when Q is a pseudodifferential operator of order zero.

RICARDO WEDER, UNAM
Inverse Scattering at a Fixed Energy

We prove that the averaged scattering solutions to the Schrödinger equation with short-range electromagnetic potentials (V, A) where $V(x) = O(|x|^{-\rho})$, $A(x) = O(|x|^{-\rho})$, $|x| \to \infty$, $\rho > 1$, are dense in the set of all solutions to the Schrödinger equation that are in $L^2(K)$ where K is any connected bounded open set in $\mathbb{R}^n$, $n \geq 2$, with smooth boundary.

We use this result to prove that if two short-range electromagnetic potentials (V_1, A_1) and (V_2, A_2) in $\mathbb{R}^n$, $n \geq 3$, have the same scattering matrix at a fixed positive energy and if the electric potentials V_j and the magnetic fields $F_j := \operatorname{curl} A_j$, $j = 1, 2$, coincide outside of some ball they necessarily coincide everywhere.

In a previous paper of Weder and Yafaev the case of electric potentials and magnetic fields in $\mathbb{R}^n$, $n \geq 3$, that are asymptotic sums of homogeneous terms at infinity was studied. It was proven that all these terms can be uniquely reconstructed from the singularities in the forward direction of the scattering amplitude at a fixed positive energy.

The combination of the new uniqueness result of this paper and the result of Weder and Yafaev implies that the scattering matrix at a fixed positive energy uniquely determines electric potentials and magnetic fields that are a finite sum of homogeneous terms at infinity, or more generally, that are asymptotic sums of homogeneous terms that actually converge, respectively, to the electric potential and to the magnetic field.

PETR ZHEVANDROV, University of Cartagena, Cartagena, Colombia
Water waves guided by underwater obstacles

It is well-known that underwater obstacles such as ridges and submerged horizontal cylinders can serve as waveguides for surface water waves. It is also known that for large values of the wavenumber k in the direction of the ridge or cylinder, there is only one guided wave. We construct the corresponding eigenfunctions and eigenfrequencies assuming that $k \to \infty$ by means of reducing the initial problem to a pair of boundary integral equations and then solving them by applying the method of Zhevandrov and Merzon (Amer. Math. Soc. Transl. (2) **208**(2003), p. 235). The resulting formulas are infinite convergent series of the Neumann type, which reduce to quite simple asymptotics of the eigenfrequencies as $k \to \infty$.