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Inverse Scattering at a Fixed Energy

We prove that the averaged scattering solutions to the Schrödinger equation with short-range electromagnetic potentials (V, A) where $V(x) = O(|x|^{-\rho})$, $A(x) = O(|x|^{-\rho})$, $|x| \to \infty$, $\rho > 1$, are dense in the set of all solutions to the Schrödinger equation that are in $L^2(K)$ where K is any connected bounded open set in $\mathbb{R}^n$, $n \geq 2$, with smooth boundary.

We use this result to prove that if two short-range electromagnetic potentials (V_1, A_1) and (V_2, A_2) in $\mathbb{R}^n$, $n \geq 3$, have the same scattering matrix at a fixed positive energy and if the electric potentials V_j and the magnetic fields $F_j := \operatorname{curl} A_j$, $j = 1, 2$, coincide outside of some ball they necessarily coincide everywhere.

In a previous paper of Weder and Yafaev the case of electric potentials and magnetic fields in $\mathbb{R}^n$, $n \geq 3$, that are asymptotic sums of homogeneous terms at infinity was studied. It was proven that all these terms can be uniquely reconstructed from the singularities in the forward direction of the scattering amplitude at a fixed positive energy.

The combination of the new uniqueness result of this paper and the result of Weder and Yafaev implies that the scattering matrix at a fixed positive energy uniquely determines electric potentials and magnetic fields that are a finite sum of homogeneous terms at infinity, or more generally, that are asymptotic sums of homogeneous terms that actually converge, respectively, to the electric potential and to the magnetic field.