Localization & Partial Differential Equations
(Org: **T. Minzoni** (IIMAS-UNAM) and **M. Ward** (UBC))

LUIS CISNEROS, Universidad Nacional Autónoma de México
A numerical study on the two-dimensional discrete sine-Gordon equation

It is known that the continuous two-dimensional sine-Gordon equation does not support radial symmetric solutions. Radial initial conditions collapse in finite time. We study numerically the discrete version of this problem, and we obtain radial symmetric solutions that do not collapse. We show using the modulation theory that the collapse is prevented by the Peierls–Nabarro potential generated by the discreteness of the problem. It is also shown that there is a threshold of radial velocities above which the Peierls–Nabarro potential is not enough to stop the collapse. The modulation theory is shown to compare favorably with the numerical solution.

JORGE FUJIOKA, Universidad Nacional Autónoma de México
Systems with Embedded Solitons

At the end of the nineties a brand-new type of solitons were discovered: *the embedded solitons*. Initially they were found in optical systems, and afterwards they were also found in hydrodynamical models, liquid crystal theory and discrete systems. These peculiar solitary waves are interesting because they exist under conditions in which, until recently, it was considered that the propagation of solitons was impossible. In the beginning it was considered that these nonlinear waves were necessarily isolated and unstable, but later on it was found that they can be stable and may exist in families. In the present communication it is explained what these *embedded solitons* are, in which models they have been found, and what variants exist (stable, unstable, continuous, discrete, *etc.*).

MARI CARMEN JORGE, Universidad Nacional Autónoma de México
A note on interface and bubble type asymptotic solutions to the two-dimensional Cahn–Hilliard equation

In this work we construct asymptotic solutions to the two-dimensional steady-state Cahn–Hilliard equation. Using variational methods we construct explicit approximate solutions of interface type. We also study the determination of the bubble type solutions inside a container with rigid walls. In this case the center of the bubble is completely determined.

THEODORE KOLOKOLNIKOV, Dalhousie University
Spot patterns in the BZ reaction

We study spot patterns in the BZ reaction. We show that these patterns can undergo two different types of self-replication. The first type of self-replication is due to fold point corresponding to a dissapearence of the steady state. The second type is due to an instability of the steady state. In the former case, the spot splits into a ring. In the latter case, the spot replicates into two spots, or forms finger-like patterns.

RACHEL KUSKE, University of British Columbia
Noise-sensitivity in bursting: new approaches for quantitative analysis

The phenomenon of bursting, composed of alternating periods of active spiking and near quiescence, is observed in a variety of biological applications including neural and cellular dynamics. It has a complex sensitivity to noise which exhibits dynamical features from both the underlying deterministic behavior and the stochastic elements. We use a combined approach of approximating both the time dependent probability density and the stochastic multi-scale dynamics in order to understand contributions from both the deterministic and stochastic features. This approach leads to simplified approximate models which can be analyzed or simulated efficiently, providing quantitative measures of the noise sensitivity. To illustrate the new approaches, we focus on a model of bursting in dendritic spines. Generalizations to other applications with similar dynamics will also be discussed.

PANAYOTIS PANAYOTAROS, IIMAS–UNAM

Localized invariant tori in the discrete NLS with diffraction management

We present results on the existence of localized invariant tori solutions in a discrete NLS equation with periodic parametric forcing. The equation models a system of coupled waveguide arrays with a special geometry that reduces diffraction effects. The solutions are obtained by continuing breather periodic solutions of an approximate autonomous system. We review some results on localized and multipeak solutions of this system and sketch a continuation argument that is based on general ideas on the continuation of invariant tori in Hamiltonian systems with symmetries.

ARTURO VARGAS, Universidad Nacional Autónoma de México, IIMAS

Evolution of Benjamin–Ono Solitons in the Presence of Weak Z-K Lateral Dispersion

The talk is about the effect of weak lateral dispersion of Z-K type on a Benjamin–Ono solitary wave. The asymptotic solution is based on an approximate variational solution for the solitary wave, which is then modulated in time through the use of conservation equations. The effect of the dispersive radiation shed as the solitary wave evolves is also included in the modulation equations. It is found that the weak lateral dispersion produces a strongly anisotropic, stable solitary wave which decays algebraically in the direction of propagation, as for the Benjamin–Ono solitary wave, and exponentially in the transverse direction. Also, it is found that the initial conditions with amplitude above a threshold evolve into solitary waves, while those with amplitude below the threshold evolve as lumps for a short time, then merge into radiation.

MICHAEL WARD, Univeristy of British Columbia, Vancouver, BC

Eigenvalue Optimization, Spikes, and the Neumann Green's Function

An optimization problem for the fundamental eigenvalue λ_0 of the Laplacian in a planar simply-connected domain that contains N small identically-shaped holes, each of a small radius $\epsilon \ll 1$, is considered. A Neumann boundary condition is imposed on the outer boundary of the domain and a Dirichlet condition is imposed on the boundary of each of the holes. For small hole radii ϵ, we derive an asymptotic expansion for λ_0 in terms of certain properties of the Neumann Green's function for the Laplacian. This expansion depends on the locations x_i, for $i = 1, \ldots, N$, of the small holes. For the unit disk, ring-type configurations of holes are constructed to optimize the eigenvalue with respect to the hole locations. This eigenvalue optimization problem is shown to be closely related to the problem of determining equilibrium vortex configurations in the Ginzburg–Landau theory of superconductivity, and is also relevant for constructing localized spike-type solutions to certain singularly perturbed reaction-diffusion systems. For these spike solutions, some stability results are also given.

RALF WITTENBERG, Department of Mathematics, Simon Fraser University, Burnaby, BC, Canada

Attractors in one-dimensional spatiotemporal chaos

We discuss the dynamics on the attractor of a family of one-dimensional PDEs displaying spatiotemporally chaotic solutions, including the Kuramoto–Sivashinsky (KS) equation. We obtain bounds and estimates on the L^2 norm and attractor dimension. A sixth-order analogue of the KS equation, the Nikolaevskii model for short-wave pattern formation with Galilean invariance,

displays a novel multiple-scale attractor. We show that existing modulation equation descriptions coupling the amplitudes for the patterned mode and mean flow, while asymptotically consistent, are incomplete. The attractor features spatiotemporal chaos with strong scale separation, coexistence of scaling regimes, anomalous exponents and Burgers-like viscous shocks. These are captured by higher-order corrections to the amplitude equations.