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Eigenvalue Optimization, Spikes, and the Neumann Green's Function

An optimization problem for the fundamental eigenvalue λ_0 of the Laplacian in a planar simply-connected domain that contains N small identically-shaped holes, each of a small radius $\epsilon \ll 1$, is considered. A Neumann boundary condition is imposed on the outer boundary of the domain and a Dirichlet condition is imposed on the boundary of each of the holes. For small hole radii ϵ, we derive an asymptotic expansion for λ_0 in terms of certain properties of the Neumann Green's function for the Laplacian. This expansion depends on the locations x_i, for $i = 1, \ldots, N$, of the small holes. For the unit disk, ring-type configurations of holes are constructed to optimize the eigenvalue with respect to the hole locations. This eigenvalue optimization problem is shown to be closely related to the problem of determining equilibrium vortex configurations in the Ginzburg–Landau theory of superconductivity, and is also relevant for constructing localized spike-type solutions to certain singularly perturbed reaction-diffusion systems. For these spike solutions, some stability results are also given.