Low-Dimensional Topology
(Org: **Victor Nuñez** (CIMAT) and **D. Rolfsen** (UBC))

LORENA ARMAS, Instituto de Matemáticas, UNAM, Circuito Exterior, Ciudad Universitaria, 04510 México DF
Paths in Open Book Decompositions

In this talk we give an algorithm to decide when a path with endpoints in the boundary of a disk with holes is simple, *i.e.*, it can be drawn without selfintersections. This algorithm is simpler than previous ones given by Zieschang, and by Matsumoto and Kamada.

This is a joint work with Francisco Gonzalez Acuña.

HANS BODEN, McMaster University
The $\mathrm{SL}(2,C)$ *Casson invariant*

We present some joint results with Cindy Curtis on the $\mathrm{SL}(2,C)$ Casson invariant for 3-manifolds. This invariant was defined by Curtis, who also established a surgery formula. Despite these results, only few computations of the invariant are known. One class of examples we explore are Seifert-fibered 3-manifolds, where we present a closed formula for the $\mathrm{SL}(2,C)$ Casson invariant which is interesting to compare with the corresponding formula for the $\mathrm{SU}(2)$ Casson. Moreover, combining these results with known results on the Culler–Shalen seminorms, we provide computations for families of 3-manifolds arising as Dehn surgeries on knots with Seifert slopes. This approach is used to investigate the behavior of the $\mathrm{SL}(2,C)$ Casson invariant for surgeries on twist knots and pretzel knots.

STEVEN BOYER, Université du Québec à Montréal, PO Box 8888, Centre-Ville, Montréal, QC H3C 3P8, Canada
On families of virtually fibred Montesinos link exteriors

William Thurston conjectured over twenty years ago that every finite volume hyperbolic 3-manifold is finitely covered by a manifold which fibres over the circle. The first non-trivial examples supporting the conjecture were obtained by Gabai and Reid. In a 1999 paper, Aitchison and Rubinstein found combinatorial conditions on certain polyhedral decompositions of 3-manifolds which guarantee the existence of such cover which fibres over the circle. In 2002, Chris Leinenger showed that every manifold obtained by Dehn filling one component of the Whitehead link exterior is finitely covered by a surface bundle and more recently Genevieve Walsh did the same for 2-bridge knot exteriors and certain Montesinos links. In this talk we show use construct several infinite families of Montesinos links which virtually fibre.

HUGO CABRERA IBARRA, Instituto Potosino de Investigación Científica y Tecnológica
Conway polynomials associated to 3-tangle closures

Given a certain type of oriented 3-string tangle, we consider five different ways for closing it to obtain knots or links and give formulas for calculating the Conway polynomials of the closures of the composition of two such 3-tangles.

MARC CULLER, University of Illinois at Chicago, MSCS (M/C 249) 851 S. Morgan St., Chicago, IL 60613, USA
Homology of small-volume hyperbolic 3-manifolds

If a closed, orientable hyperbolic 3-manifold M has volume less than 1.219 then $H_1(M\,;Z)$ has rank at most 3. Moreover, unless M is an exceptional manifold in the sense of Gabai, Meyerhoff and N. Thurston, the rank of $H_1(M\,;Z_p)$ is at most

2 for any odd prime p. There are three examples of manifolds known with volume less than 1.219, one of which, namely the Weeks manifold, has mod 5 first homology of rank 2. The proof combines several deep results about hyperbolic 3-manifolds, including the work of Gabai–Meyerhoff–Thurston on maximal tube radius; the Marden Tameness Conjecture, proved by Agol and Calegari–Gabai; the $\log(2k-1)$ Theorem, proved with Anderson, Canary and Shalen; and bounds on volume change under Dehn filling obtained by Agol, Dunfield, Storm and W. Thurston using results from Perelman's work on Ricci flow. The basic strategy is to compare the volume of a tube about a shortest closed geodesic C in M with the volumes of tubes about closed geodesics in a sequence of hyperbolic manifolds obtained from M by Dehn surgeries on C.

This is joint work with Ian Agol and Peter Shalen.

MARIO EUDAVE, Instituto de Matemáticas, UNAM, Circuito Exterior, Ciudad Universitaria, 04510 México DF
Some examples of knots with interesting properties

In this talk we give some explicit examples of knots or links which give a positive answer to questions made by several people. First, for each odd number n, we find knots whose exterior contains a connected, orientable, incompressible surface with n boundary components, answering a question of F. Gonzalez-Acuña and A. Ramirez. Second, we construct knots whose exterior contains an incompressible torus with four boundary components, and a non-meridional slope; these examples are simpler that the ones given by the author some years ago, which answered a question made by J. Luecke. Third, for each rational number p/q, we find a link with two components $k_1 \cup k_2$, so that by doing p/q-Dehn surgery on k_1 gives a reducible manifold, answering a question of N. Sayari.

WOLFGANG HEIL, Florida State University
3-manifolds covered by three open balls or solid tori

Hempel and McMillan showed that a closed 3-manifold M that can be covered by three open balls is a connected sum of S^3 and S^2-bundles over S^1. We sketch a new proof of a slightly generalized result and show how the proof can be adapted to classify closed 3-manifolds covered by one open solid torus and two open balls or by two open solid tori and one open ball. We think that with these methods we will be able to handle the case for 3 open solid tori.

GABRIEL INDURSKIS, University of British Columbia, Dept. of Mathematics, #121-1984 Mathematics Road, Vancouver, BC, V6T 1Z2, Canada
Exceptional fillings of once-punctured torus bundles

Let M be a hyperbolic 3-manifold which is a bundle over the circle with a once-punctured torus as fibre. Its monodromy is conjugate in $\mathrm{SL}(2,\mathbb{Z})$ to the canonical form $\pm R^{a_1}L^{b_1}\cdots R^{a_n}L^{b_n}$ with positive exponents, where $n>0$ and R and L are the upper and lower triangular matrices generating $\mathrm{SL}(2,\mathbb{Z})$. We show that when $n>5$, there is only one non-hyperbolic Dehn filling of the bundle (namely the Dehn filling with slope isotopic to the boundary of the fibre). This concretizes a result of Bleiler and Hodgson which showed the existence of such a lower bound. The bound is sharp, as there are bundles with $n=5$ which admit two exceptional fillings.

This is joint work with David Futer (Michigan State University).

EDGAR JASSO, UNAM
On knots with Seifert Fibered Dehn Surgeries

For a hyperbolic knot in S^3 there are only finitely many Dehn surgeries yielding non-hyperbolic manifolds. In this talk we will present an infinite family of (hyperbolic) knots admitting a Seifert-fibered Dehn surgery; these knots do not arise from the primitive/Seifert-fibered construction introduced by J. Dean a few years ago. The pretzel knot $(3,-3,3)$ belongs to this family.

This is joint work with Mario Eudave-Muñoz.

MERCEDES JORDAN-SANTANA, ICTP
A geometric proof that the singular braid group is torsion-free

Singular Braids can be seen as the classical braids where we allow two strings to intersect. If the intersecting points are marked with white or black colour we can give the definition of life-disc in order to create a group, the singular braid group. In this talk we give the presentation of this group, its geometric description and some properties that help us to prove that this group is torsion-free.

CHRISTIAN LAING, Florida State University, 208 Love Building, Tallahassee, Florida USA 32306-4510
Geometric Measures as Brain Shape Descriptors

Classification and identification of differences in brain anatomy (*i.e.*, differences in shape) can play an important role in Neuroscience. Methods such as Magnetic Resonance Imaging (MRI) are used to correlate brain structure and function, and to measure changes during development and disease.

Given a set of polygonal curves (not necessarily closed or connected), geometric measures involving combinations of writhe and average crossing numbers of subcurves, as well as ropelength and thickness, can be computed to obtain a set of features for the purpose of shape characterization. These measures, originally given for simple closed curves, can be defined in a natural way for a set of polygonal curves.

We apply these geometric measures to a set of curves obtained by tracing sulcal paths on the gray matter surface of human brains. These surfaces are extracted from MRI scans of human brains. We then compute these geometric measures to construct a feature vector which is used in a machine learning process. A clustering technique called multiple discriminant analysis is used to find an optimal projection of the feature space into a plane. This optimal planar projection minimizes the variance within a cluster, and maximizes the distance between distinct clusters.

In our preliminary results, an automatic differentiation between sulcal paths from the left or right hemispheres was possible. Also a male-female classification and younger-older classification was achievable.

J. CARLOS GÓMEZ LARRAÑAGA, CIMAT
Lusternik–Schnirelmann type invariants for 3-Manifolds

We will talk about what is known about these invariants for 3-Manifolds.

MAX NEUMANN, Instituto de Matemáticas, UNAM
Surfaces in semi-alternating knot complements

I want to consider knots made by glueing two alternating tangles (with few strings or with some other conditions) and look for essential surfaces in their complements.

VICTOR NUÑEZ, Cimat, AP 402, Guanajuato, Gto., México
Coverings of Montesinos knots of the second kind

A Montesinos knot $k \subset S^3$ is a link that has a Seifert manifold as double branched covering, $B_2(k)$, and such that k is not a union of fibers of a Seifert fibering of the 3-sphere (this is not an exact definition, but the exceptions are few and non-interesting for us).

For the first kind of Montesinos knots, the double branched covering is an orientable Seifert manifold with orbit surface the 2-sphere. A Montesinos knot k is of the second kind, if $B_2(k)$ is an orientable Seifert manifold with orbit surface a non-orientable surface.

For a Montesinos knot k of the first kind, it is known how to get the 3-sphere as a dihedral branched covering of (S^3, k). We explain how to obtain the 3-sphere as a dihedral covering, and also as a 'meta-dihedral' branched covering of (S^3, k). These last 'meta-dihedral' coverings show a surprising and very interesting similarity with the study of Montesinos knots of the first kind.

Joint work with Jair Remigio.

ENRIQUE RAMIREZ, CIMAT

There exist infinitely many two component links which are 2-universal

A link or knot l is 2-universal if every closed orientable 3-manifold is a covering of S^3 branched along l, and each branching index is either one or two. In this work we give a family of 2-universal links.

DALE ROLFSEN, University of British Columbia, Vancouver, BC V6T 1Z2, Canada

On R-covered foliations of 3-manifolds

A codimension one foliation of a manifold is said to be R-covered if the space of leaves of the pullback of the foliation to the universal cover is homeomorphic to the real line. Sometimes the definition also requires that the foliation of the manifold be transversely oriented, but we will not assume this.

We will present a family of 3-manifolds which possess R-covered foliations, but on the other hand cannot be given a foliation which is both transversely oriented and R-covered. We use a theorem of Calegari and Dunfield that if a 3-manifold has a transversely oriented R-covered foliation, then its fundamental group is left-orderable. Our examples are Haken manifolds which have finite homology groups, possess R-covered foliations, but have non-left-orderable fundamental groups.

MARTY SCHARLEMANN, University of California, Santa Barbara

Straightening tube sums

Suppose the complement of a 3-manifold $W \subset S^3$ contains an incompressible torus T. Then there is a natural way to reimbed W in a typically simpler form, namely reimbed the solid torus that T bounds in S^3 as an unknotted solid torus. When T is of higher genus there is not such a natural choice of reimbedding. For the special genus two case in which T is a tubed sum of two distant tori there is a natural choice. We show how this case suffices as a 3-dimensional tool to prove the genus three, 4-dimensional Schoenflies Conjecture.

DEWITT SUMNERS, Florida State University

DNA Knots Reveal Chiral Packing of DNA in Phage Capsids

Bacteriophages are viruses that infect bacteria. They pack their double-stranded DNA genomes to near-crystalline density in viral capsids and achieve one of the highest levels of DNA condensation found in nature. Despite numerous studies some essential properties of the packaging geometry of the DNA inside the phage capsid are still unknown. Although viral DNA is linear double-stranded with sticky ends, the linear viral DNA quickly becomes cyclic when removed from the capsid, and for some viral DNA the observed knot probability is an astounding 95%. This talk will discuss comparison of the observed viral knot spectrum with the simulated knot spectrum, concluding that the packing geometry of the DNA inside the capsid is non-random and writhe-directed.

References

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MARIEL VAZQUEZ, San Francisco State University, Mathematics Department, 1600 Holloway Ave, San Francisco, CA 94132, USA

Processes of topology simplification in biology

Important Biological processes such as replication, transcription and recombination involve global topological changes of long DNA molecules. Circular DNA adopts different topological conformations in the cell, negative supercoiling being its preferred, native state. There is evidence that knots inhibit replication and transcription, and it is known that links with two or more components prevent proper segregation at cell division. The cell has thus devised ways to reduce topological entanglement. I will talk about recent models of DNA unknotting and unlinking both from a biological, a mathematical and a computational point of view.

This is joint work with J. Arsuaga, I. Grainge, X. Hua, D. Sherratt and S. Trigueros.

GENEVIEVE WALSH, University of Quebec at Montreal and Tufts University

Commensurability classes of two-bridge knot complements

Two 3-manifolds are said to be commensurable if they have a common finite-sheeted cover. Commensurability classes are a reasonable way to organize hyperbolic 3-manifolds. For example, if a manifold is virtually fibered or virtually Haken, then so is every manifold in its commensurability class. However, the general problem of determining if two hyperbolic 3-manifolds are commensurable is difficult. We show that a hyperbolic 2-bridge knot complement is the unique knot complement (in S^3) in its commensurability class. The proof relies heavily on facts particular to 2-bridge knots.

There are commensurability classes that contain more than one hyperbolic knot complement. For example, this can happen if one of the knots admits a lens space surgery. We speculate on the general case.

This is joint work with Alan Reid.