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Homology of small-volume hyperbolic 3-manifolds

If a closed, orientable hyperbolic 3-manifold M has volume less than 1.219 then $H_1(M; \mathbb{Z})$ has rank at most 3. Moreover, unless M is an exceptional manifold in the sense of Gabai, Meyerhoff and N. Thurston, the rank of $H_1(M; \mathbb{Z}_p)$ is at most 2 for any odd prime p . There are three examples of manifolds known with volume less than 1.219, one of which, namely the Weeks manifold, has mod 5 first homology of rank 2. The proof combines several deep results about hyperbolic 3-manifolds, including the work of Gabai–Meyerhoff–Thurston on maximal tube radius; the Marden Tameness Conjecture, proved by Agol and Calegari–Gabai; the $\log(2k-1)$ Theorem, proved with Anderson, Canary and Shalen; and bounds on volume change under Dehn filling obtained by Agol, Dunfield, Storm and W. Thurston using results from Perelman’s work on Ricci flow. The basic strategy is to compare the volume of a tube about a shortest closed geodesic C in M with the volumes of tubes about closed geodesics in a sequence of hyperbolic manifolds obtained from M by Dehn surgeries on C .

This is joint work with Ian Agol and Peter Shalen.