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Cycles in sparse graphs

The central theme of this talk is to study the largest possible average degree $d(n, S)$ of an n-vertex graph with no cycle of length from a given set of positive integers S.

When S contains only odd numbers, the extremal graphs are complete bipartite graphs, so $d(2n, S) = n$ in this case. When S contains even numbers, the problem becomes notoriously difficult. The case $S = \{2k\}$ is Erdős' Even Cycle Theorem. In this talk I will give a short proof of this theorem, which states $d(n, S)$ is at most about $n^{1/k}$. The method used to prove this allows us to consider any set S of forbidden even cycle lengths: a general theorem will be presented which shows that apart from chaotic looking sets S, $d(n, S)$ is at most about $\exp(\log^* n)$. This is motivated by a conjecture of Erdős that $d(n, S)$ is a constant when S is the set of powers of two. Very surprisingly, our result is tight: there exist sets S for which $d(n, S)$ is roughly $\exp(\log^* n)$.