ISIDORO GITLER, Department of Mathematics, Cinvestav
On a new class of Mengerian hypergraphs

We study the normality of the Rees algebra associated to a clutter and the relations with the conjecture of Conforti and Cornuéjols about the packing properties on clutters. This conjecture states that all the clutters with the packing property have the MFMC property. We show that this conjecture is equivalent to an algebraic statement about the normality of the Rees algebra. Finally we introduce a new infinite class of hypergraphs that verify the conjecture of Conforti and Cornuéjols.

Definition 1 The clutter $\mathcal{C}$ satisfies the *max-flow min-cut* (MFMC) property (or is called Mengerian) if both sides of the LP-duality equation

$$\min\{\langle\alpha, x\rangle \mid x \geq 0 \,;\, xA \geq \mathbf{1}\} = \max\{\langle y, \mathbf{1}\rangle \mid y \geq 0 \,;\, Ay \leq \alpha\} \tag{1}$$

have integral optimum solutions x and y for each non-negative integral vector α.

We denote the smallest number of vertices in any minimal vertex cover of $\mathcal{C}$ by $\alpha_0(\mathcal{C})$ and the maximum number of independent edges of $\mathcal{C}$ by $\beta_1(\mathcal{C})$.

Definition 2 If $\alpha_0(\mathcal{C}) = \beta_1(\mathcal{C})$ we say that the clutter $\mathcal{C}$ has the König property.

Definition 3 A clutter $\mathcal{C}$ satisfies the *packing property* (PP) if all its minors satisfy the König property.

Proposition 1 *If $\mathcal{C}$ has the max-flow min-cut property, then $\mathcal{C}$ has the packing property.*

Conjecture 1 (Conforti-Cornuéjols) If the clutter $\mathcal{C}$ has the packing property, then $\mathcal{C}$ has the max-flow min-cut property.

This conjecture continues to be open. The only cases known are for binary matroids (Seymour), and dyadic hypergrphs (Cornuejols, Guenin and Margot).

In this talk we give a new infinite family Q^F_{pq} of hypergrahs that verify the conjecture of Conforti and Cornuéjols. This new class is neither binary nor dyadic.