Functional Analysis
(Org: **Hugo Arizmendi** (UNAM), **Anthony Lau** (Alberta) and **Lourdes Palacios** (UAM))

LINE BARIBEAU, Université Laval, Québec (QC), Canada

The spectral Schwarz lemma revisited

An algebroid function $K(z)$ is the set-valued function obtained by taking the zeroes of a polynomial whose coefficients are holomorphic functions of z. We present a sharpened version of the Schwarz lemma for algebroid functions, and discuss it in the context of the spectral Nevanlinna–Pick problem.

JONATHAN BORWEIN, Dalhousie University

Maximality of Sums of Monotone Operators

We say a multifunction $T\colon X \mapsto 2^{X^*}$ is *monotone* provided that for any $x, y \in X$, and $x^* \in T(x)$, $y^* \in T(y)$,

$$\langle y - x, y^* - x^* \rangle \geq 0,$$

and that T is *maximal monotone* if its graph is not properly included in any other monotone graph. The *convex subdifferential* in Banach space and a *skew linear matrix* are the canonical examples of maximal monotone multifunctions. Maximal monotone operators play an important role in functional analysis, optimization and partial differential equation theory, with applications in subjects such as mathematical economics and robust control. In this talk, based on [1], I shall show how—based largely on a long-neglected observation of Fitzpatrick —the originally quite complex theory of monotone operators can be almost entirely reduced to convex analysis. I shall also highlight various long standing open questions which these new techniques offer new access to.

References

[1] J. M. Borwein, *Maximal Monotonicity via Convex Analysis.* J. Convex Analysis (Special issue in memory of Simon Fitzpatrick) **13** (June 2006). [D-drive Preprint 281].

CARLOS BOSCH, Instituto Tecnólogico Autónomo de México

Multipliers of Temperate Distributions

We will show that the space Oq of multipliers of temperate distributions can be expressed as the inductive limit of certain Hilbert spaces.

Joint work with Jan Kucera.

SLAVISA DJORJEVICH, Facultad de Ciencias Físico–Matemáticas, BUAP, Apdo. Postal 1152 Puebla, Pue. 72000

Spectrum of Upper Triangular Operator Matrices

Let H and K be Banach spaces, let $B(H,K)$ denote the set of bounded linear operators from H to K, and abbreviate $B(H,H)$ to $B(H)$. For the operators $A \in B(H)$, $B \in B(K)$ and $C \in B(K,H)$, let M_C denote the operator matrices in

$B(H \oplus K)$ defined with

$$M_C = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix} : H \oplus K \to H \oplus K. \tag{1}$$

In this talk we will describe spectrum, Weyl's and Browder's spectrum of operator matrices M_C using spectral property of operators A and B.

SALVADOR PÉREZ ESTEVA, Instituto de Matemáticas Cuernavaca, Universidad Nacional Autónoma de México, Av. Universidad sn, Lomas de Camilpa, Cuernavaca México CP 62251
Atomic decompositions in Banach-valued Hardy spaces on Lipschitz domains

We prove an atomic decomposition for all the Borel measures that arise as boundary limits of Banach-valued harmonic functions on a Lipschitz domain D, whose non-tangential maximal function is integrable with respect to harmonic measure of the boundary of D. As in the case of the disk, the existence of non-tangential boundary limits of all these harmonic functions characterizes the Radon–Nikodym property of the Banach space.

FERNANDO GALAZ, CIMAT, UAM–Iztapalapa
Iterating the Cesaro operator

Given a complex sequence $s = \{a_n\}$, the discrete Cesaro operator T assigns to it the sequence $Ts = \{b_n\}$, where $b_n = \frac{a_0+\cdots+a_n}{n+1}$, $n = 0, 1, \ldots$. If s is a convergent sequence, we prove that $\{T^n s\}$ converges if, and only if, $a_1 = \lim_{n\to\infty} a_n$. We also establish a corresponding result for the continuous Cesaro operator defined on $C[0, 1]$.

BERTA GAMBOA DE BUEN, Centro de Investigación en Matemáticas
Empty intesection of slices and the fixed point property in Banach spaces

We prove that the condition of the empty slice property (EIS), which is a generalization of uniform smoothness, implies the fixed point property. That is, in a Banach space with EIS, every nonexpansive map from a weakly compact convex set into itself has a fixed point. Furthermore, the EIS property is stable under finite l^p sums of Banach spaces. We also give some examples.

(Joint work with Helga Fetter)

ARMANDO GARCÍA, Universidad Nacional Autónoma de México, Instituto de Matemáticas, UNAM, Circuito Exterior, Ciudad Universitaria
An extension of Ekeland's variational principle to locally complete spaces

We prove an extension of Ekeland's variational principle to locally complete spaces which uses subbaditive, strictly increasing continuous functions as perturbations.

ZHIGUO HU, University of Windsor, Windsor, Ontario N9B 3P4, Canada
Multipliers and topological centre problems

We present some recent results on multipliers of a Banach algebra and their applications to topological centre problems.

The talk is based on joint work with Matthias Neufang and Zhong-Jin Ruan.

MICHAEL LAMOUREUX, Dept. of Mathematics and Statistics, University of Calgary
Linear operators and minimum phase

Geophysical applications demand a mathematical modeling of physical processes that respect minimum phase conditions. Essentially, this states that energy in a signal is concentrated near the beginning of the onset of a signal. We present a mathematical definition of minimum phase, develop robust calculation of equivalent minimum phase signals, and examine the class of linear operators on Hilbert space that preserve minimum phase. Properties are closely connected to factorization problems in Hardy space.

ALEXANDER LIVAK, University of Alberta, Edmonton, AB T6G 2G1, Canada
A covering lemma and its applications

An entropy lemma states that if we control the diameter of a body on a subspace then we control the covering of the body. More precisely, given two centrally-symmetric bodies K and L, satisfying $K \subset AL$ and $K \cap E \subset aL$ for a k-codimensional subspace E, one has $N(K, 2rL) \leq \big(4A/(r-a)\big)^k$ for every $r > a$. That means that, surprisingly, the covering numbers of K behave in the same way as the covering numbers of a cylinder with the base $aL \cap E$. We prove this lemma and discuss its applications to the Gelfand numbers and to the Sudakov inequality.

This talk is based on joint works with A. Pajor and N. Tomczak-Jaegermann and with V. Milman, A. Pajor, and N. Tomczak-Jaegermann.

LAURENT MARCOUX, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Sums of small numbers of commutators

For many C^*-algebras $\mathcal{A}$, techniques have been developed to show that all elements which have trace zero with respect to all tracial states can be written as a sum of finitely many commutators, and that the number of commutators required depends only upon the algebra, and not upon the individual elements. In this paper, we show that if the same holds for $q\mathcal{A}q$ whenever q is a "sufficiently small" projection in $\mathcal{A}$, then every element that is a sum of finitely many commutators in $\mathcal{A}$ is in fact a sum of two. We use these results to show that many C^*-algebras are linearly spanned by their projections.

RUBÉN MARTÍNEZ AVENDAÑO, Universidad Autónoma del Estado de Hidalgo, Centro de Investigación en Matemáticas, Ciudad Universitaria, Pachuca, Hidalgo, México
Eigenmatrices and operators commuting with finite-rank operators

Using eigenmatrices, we characterize when a bounded operator in Hilbert space commutes with a finite-rank operator. We use this characterization to prove that if an operator commutes with a finite-rank operator, then it must commute with an operator of rank one. As a corollary of this, we show that (classical) Toeplitz operators do not commute with operators of finite rank.

MEHDI SANGANI MONFARED, University of Windsor, Department of Mathematics and Statistics, 401 Sunset Avenue, Windsor, ON N9B 3P4
Character Amenability of Banach Algebras

The notion of character amenability of Banach algebras will be discussed. It will be shown that for a locally compact group G, the amenability of either of the group algebra $L^1(G)$ or the Fourier algebra $A(G)$ is equivalent to the amenability of the underlying group G.

We also discuss some cohomological implications of character amenability. In particular we show that if A is a commutative character amenable Banach algebra, then $\mathcal{H}^n(A,E)=\{0\}$ for all finite-dimensional Banach A-bimodules E, and all $n\in\mathbb{N}$. This in particular implies that all finite-dimensional extensions of such Banach algebras split strongly. This extends earlier results of H. Steiniger and myself on Fourier and generalized Fourier algebras to the larger class of commutative character amenable Banach algebras.

MATTHIAS NEUFANG, School of Mathematics and Statistics, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario K1S 5B6, Canada

Quantum groups and quantum information theory

Let $\mathbb{G}=(\mathcal{M},\Gamma,\varphi,\psi)$ be a co-amenable locally compact quantum group. In recent work with M. Junge and Z.-J. Ruan, we have constructed and studied a completely isometric representation of the algebra of completely bounded (right) multipliers of $L_1(\mathbb{G})$ on $\mathcal{B}\big(L_2(\mathbb{G})\big)$. This extends and unifies earlier work by F. Ghahramani, E. Størmer and myself in the case $\mathcal{M}=L_\infty(G)$, and by Z.-J. Ruan, N. Spronk and myself for $\mathcal{M}=VN(G)$, where G is a locally compact group. We have shown that the multiplier algebra can in fact be identified with the algebra of completely bounded, normal $\widehat{\mathcal{M}}$-bimodule maps on $\mathcal{B}\big(L_2(\mathbb{G})\big)$ leaving $\mathcal{M}$ invariant. The part of the latter algebra consisting of completely positive maps provides a natural class of quantum channels which, from the viewpoint of quantum computing, are of particular interest in the case of finite-dimensional quantum groups.

In this talk, we shall discuss various applications of the above representation to quantum information theory. Indeed, several properties of our channels are highly desirable with regard to quantum error correction: the bimodule property means precisely that the channels are noiseless for $\widehat{\mathcal{M}}$; moreover, every such channel has a symbol which is easy to retrieve, and the completely bounded minimal entropy (*cb-entropy*) can be calculated explicitly. Note that the cb-entropy has recently be shown to be additive (I. Devetak, M. Junge, C. King, and M. B. Ruskai); proving additivity of the bounded minimal entropy is a major open problem in quantum information theory.

This is joint work with Marius Junge, David Kribs and Zhong-Jin Ruan.

NICO SPRONK, University of Waterloo, Waterloo, Ontario

The algebra generated by idempotents in a Fourier-Stieltjes algebra

Let G be a locally compact group. The Fourier–Stieltjes algebra $B(G)$ is the dual space of the group C^*-algebra $C^*(G)$, and it can be naturally be made into Banach algebra which can be identified with a subalgebra of the bounded continuous functions on G. If G is abelian, then $B(G)$ is exactly the algebra of Fourier–Stieltjes transforms of measures on the dual group. As such, $B(G)$ is a large commutative Banach algebra and, frequently, has an intractable spectrum and is not regular.

We consider the closed span of the idempotents in $B(G)$, $B_I(G)$. Even for totally disconnected groups, $B_I(G)$ is a regular Banach algebra. M. Ilie and I have computed the spectrum of $B_I(G)$, and characterised, for another locally compact group H, when $B_I(G)$ is isometrically algebraically isomorphic to $B_I(H)$. We have also computed some examples. This represents an application of the "spine" of $B(G)$, which we defined previously, and has a nice application in amenability theory.

VLADIMIR TROITSKY, University of Alberta

Minimal vectors and invariant subspaces

The method of minimal vectors was developed to find invariant subspaces of certain classes of operators on Hilbert spaces. We describe applications of this method to Banach spaces, Banach lattices, and algebras of operators.

ANTONI WAWRZYNCYK, Universidad Autónoma Metropolitana–Iztapalapa, Departamento de Matemáticas, Av. San Rafael Atlixco 186, Col. Vicentina, 09340 Mexico, AP 55-534
Schur Lemma and the spectral mapping formula

Let B be a complex topological unital algebra. The left joint spectrum of a set $S \subset B$ consisting of pairwise commuting elements is defined by the formula

$$\sigma_l(S) = \Big\{(\lambda(s))_{s\in S} \in \mathbb{C}^S \;\Big|\; \sum_{s\in S} B(s - \lambda(s)) \text{ is a proper ideal}\Big\}.$$

Using the Schur Lemma and the Gelfand–Mazur theorem we prove that $\sigma_l(S)$ has the spectral mapping property for the following algebras:

(i) B—a locally convex (F)-algebra with all maximal left ideals closed,

(ii) B—an m-convex algebra with all maximal left ideals closed,

(iii) B—a locally convex Waelbroeck algebra.

The right ideals version of the result is also valid.