JUAN RADA, Unversidad de Los Andes

A generalization of the energy to digraphs

The adjacency matrix $A = (a_{ij})$ of a graph G with set of vertices $\{v_1, \ldots, v_n\}$ and set of edges E_G is defined as

$$a_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E_G \\ 0 & \text{if } v_i v_j \notin E_G. \end{cases}$$

The eigenvalues of the graph G are the eigenvalues of the adjacency matrix A. Since A is real and symmetric, the eigenvalues $\lambda_1, \ldots, \lambda_n$ of G are real numbers. The energy of G, denoted by $E(G)$, is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

One of the long-known results in this field is the Coulson integral formula. In this article, we extend the concept of energy to directed graphs in such a way that Coulson Integral Formula remains valid. As a consequence, it is shown that the energy is increasing over the set $\mathcal{D}_{n,h}$ of digraphs with n vertices and cycles of length h, with respect to a quasi-order relation. Applications to the problem of extremal values for the energy in various classes of digraphs are considered.