REPORT OF WORKING GROUP 6
MATHEMATICAL MODELLING AND SCIENCE

Co-Leaders:

France	Caron	Université de Montréal
Hassane	Squalli	Université de Sherbrooke

Participants:

Jacques	Bélair	Université de Montréal
Rick	Brewster	Thompson Rivers University
Hugh	Burkhardt	University of Nottingham
Egan	Chernoff [*]	University of Saskatchewan
George	Ekol [*]	Simon Fraser University
Philippe	Etchecopar	Cegep de Rimouski
Kevin	Keen[*]	University of Northern British Columbia
Steven	Khan	University of British Columbia
Marc	Laforest	École Polytechnique
Randall	Pyke	Simon Fraser University
Deanna	Shostak	Alberta Education
Jean-Philippe	Villeneuve	Cegep de Rimouski

This working group was set up at the beginning of 2008 from the following project proposals which had been submitted for the Forum:

- *High School Mathematical Modeling Contest* (J. Bélair)
- *Intégration de la démarche de modélisation-simulation et de l'environnement dans l'enseignement des mathématiques* (P. Etchecopar and J.P. Villeneuve)
- *A Canadian web portal promoting applications of mathematics in engineering, biology and the social science* (M. Laforest)
- *Les approches interdisciplinaires en mathématiques, science et technologie* (H. Squalli)

These projects all shared a common vision: the development of mathematical modelling (and not simply the teaching of models) should be considered one of the goals of mathematics education, and support should be provided to teachers to assist them in contributing to that goal.

At the forum in Vancouver, the group spent the first day in Vancouver sharing and discussing different activities aimed at enriching the modelling component of the mathematics curriculum, from secondary to university.

In the first session of the second day, the group met with the Significant Statistics group. As it became clear that the two groups had several common goals for which they could join forces, they pursued the next session together.

[*] Members of the "Significant Statistics" working group.

Looking at modelling activities

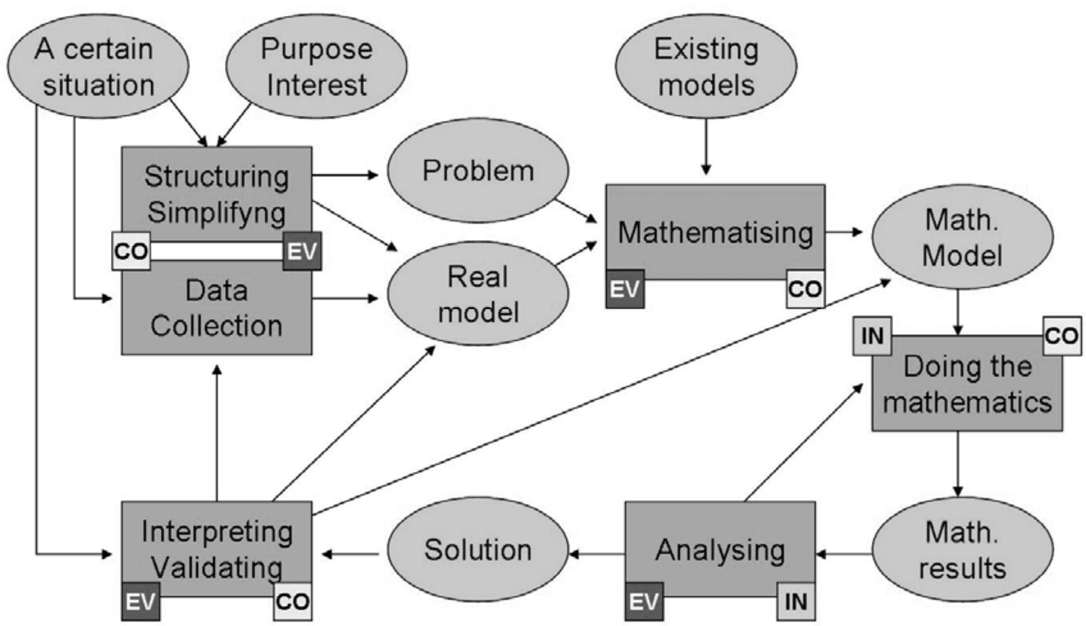
In looking at the modelling activities, the value of developing skills in modelling as one of the goals of mathematics education was reaffirmed, both for its social relevance and potential contribution to students' motivation and mathematical thinking: it helps students gain awareness that mathematics is a precious tool for understanding the world in which they live.

The proposed modelling activities, which can be found in the appendix to this report, had been characterized with respect to

1. the mathematics that can be used to model the situation
2. the phases of the modelling cycle that warrant greater attention
3. the modelling skills that they can help develop.

This was done using the following diagram, which was built from the description of the modelling cycle provided in the discussion document for ICMI Study 14 on Applications and Modelling in Mathematics Education (Blum et al., 2002). Each of the phases of the cycle was associated with the types of skills it mainly requires or could help develop. For this, we used the following classification of competencies proposed by De Terssac (1996):

- *communication skills* (CO) : to translate, represent, interpret what the context is, what is to be done and what has been done.
- *intervention skills* (IN) : to act upon a situation by using available knowledge and by transforming encountered situations into reusable knowledge;
- *evaluation skills* (EV) : to identify, choose and justify whatever is being or has been engaged into action.



The question was raised as to whether or not we use communication skills in the phase where we "do the mathematics". Strictly speaking, mathematics is always "done" using some form of language (including software commands and programming languages for complex problems). Yet, when introducing modelling in school mathematics, it is in getting an understanding of the situation and the goal pursued, and in sharing the results and their interpretation, that communication skills play an unprecedented role. In authentic professional modelling activities, these phases of communication are typically done in interaction with a client or an expert in the situation to be modelled.

Tackling complexity

The presentation and discussion of the various modelling situations raised the question of the nature and degree of complexity of the situation proposed: At what scale should the situation be approached? What variables should we be looking at? Is the phenomenon sensitive to initial conditions? How authentic, precise and reliable is the data? Should we account for natural variability? How precise should the results be? In addressing these questions, a consensus emerged to the effect that the same situation could be the object of modelling at different school grade levels, with an increasing degree of sophistication in the definition of the problem and the model. For instance, the operational duration of a landfill site could be modelled very simply by considering only its capacity and the volume of waste it receives each year. Finer models would include, in ascending order of sophistication: compression coefficients, ideally distinct for different strata, composition of each stratum, gas flow, etc. Accounting for variability can lead to probabilistic modelling, which is known to be more challenging for students than deterministic modelling. Ill-structured situations lend themselves well to developing autonomy in making modelling decisions.

As was suggested by the presentation of the activities, modelling can be taught by focusing initially on small parts of the modelling cycle. Critical analysis can and should lead to iterative improvement of the models. The process of measuring the validity of a model is the essential activity towards which teachers should strive, but it is also the activity which requires a mastery of the entire modelling cycle.

Technology is considered essential in tackling the complexity of real situations, first for solving associated problems, but also, and increasingly, for gathering data and developing models. However, letting students use technology for solving relatively complex problems that they could not approach with the pen and paper techniques that they have been taught, may lead to a tension between a possible gain in intervention skills (with the expansion of the class of problems that they can now solve) and a possible loss in evaluation skills (as the black boxes used may increase in size and number). Beyond the necessary knowledge of the limitations of the tools or software used, this may call for a more systematic study within the math curriculum of the alternate (rather than new…) techniques enabled by technology.

Principles of modelling education

As participants discussed the various modelling activities and implications for teaching, a set of principles for modelling education emerged:

- The problems should be open-ended, allowing for different strategies and solutions.

- We should develop in the math classroom culture a tolerance to uncertainty and strategies for learning to deal with it.
- In the same way that we have learned to value the use of multiple representations for teaching and learning a given mathematical concept, we should also value different types of models (equations, graphics, diagrams, graphs, closed-form functions, differential equations, recurrence relations, matrix representations, etc.) as multiple representations of a given situation (or of one of its components) with possibly distinct advantages: efficiency, generality, precision, readability, convincingness, explanatory power, etc.
- We should be careful about not reducing the assessment of modelling to assessing knowledge and use of “the” model that has been taught for a given type of situation.
- Acknowledging that modelling takes time, we should be looking for different forms of assessment than traditional exams.
- Modelling typically is an interdisciplinary and collective process, which takes advantage of different perspectives and expertise. We should look for ways to have this reflected in some of the learning activities and assessment tasks.
- Many people argue that modelling should occur exclusively in science classes, and not in mathematics classes; but the fact is that generally, modelling is rarely addressed in either, beyond the teaching of predefined models. In addition, as some of the most accessible modelling activities may fall outside the school science curriculum, and as modelling should become part of the way we address daily life situations, the group argues that mathematical modelling should be learned, not exclusively but in large part, in mathematics classes.

Using modelling to teach mathematics

Mathematical modelling can be seen both as a learning objective in its own right and as a means to learning mathematics. Although these two objectives are not mutually exclusive, balancing them in the teaching of mathematics is not a straightforward task (Blum et al., 2002). The group brought some ideas to help reach an appropriate balance.

While keeping in mind that not every lesson of mathematics has to include modelling, it was clear for the group that modelling in school mathematics should not be reduced to curve fitting activities. Not only do such activities contribute little to students’ understanding of either the situation or the mathematics used (as technology is typically responsible for handling regression), they often present situations where reality has been substantially distorted to match the functions that have just been taught. The way tides are typically presented in the high school math curriculum is an example of such distortion.

A teacher may create the need for mathematical concepts that have yet to be taught by proposing problems that can be partially (or less efficiently) tackled with students’ current mathematical toolkit. Not only does such an approach contribute to the development of modelling and problem solving, it also favours anticipation of new mathematical concepts with an intuition of their meaning or properties, and can help establish connections between concepts and techniques.

Modelling can also be performed within a pure mathematical context. For instance, in addition to visualisation of classical geometry optimisation problems, dynamic geometry environments enable observation of change and identification of loci, properties, patterns and invariants. Furthermore, remaining within the mathematical world may offer the possibility of using deductive reasoning to build a model or validate a solution. Despite the fact that it bypasses the complexity of real situations, modelling within a mathematical context may reveal an interesting intermediate approach, which has some value on its own and can also prepare the ground for more interdisciplinary modelling.

Resources for modelling education

Although they may not always be easy to find, or readily usable, resources for mathematical modelling education are multiplying. The access we have gained through internet to statistics of all sorts (including health and environmental data), documented academic or industrial experiences, and various types of archives, makes it more feasible to use genuine data and authentic questions, thereby enabling more meaningful experience in the modelling projects and activities. Yet, there is still a need to provide more direct access for teachers to these resources and to document modelling activities which can be built upon them. Professional modellers can be key players in the process, but classroom usability of such activities heavily depends on the involvement of educators.

Despite a lack of tradition, collaborations between teachers of different disciplines can reveal beneficial to both teachers and students. One approach that was proposed to operationalize at the school level the way modelling is performed in the "real world" is to have a teacher of another discipline act as "client" for a modelling project: not only can such client describe the situation, some of its underlying principles, and the goal to be achieved through modelling, he or she can also participate in validating the model and assessing student's performance in approaching the stated goal.

Additional teacher support avenues

Concrete measures were proposed to assist teachers in integrating a modelling component to their teaching of mathematics. From our session with the "Significant Statistics" working group, it became very clear that the integration of modelling in the teaching of mathematics and the development of statistical thinking had a lot to share, and could really build on common grounds.

As a first proposal, and possibly one of the most significant outcomes of the meeting, the idea of having a national modelling contest for secondary school students evolved as a possible joint initiative from the Canadian Applied and Industrial Mathematics Society (CAIMS) and Statistical Society of Canada (SSC).

Other proposals included the development of workshops that would be functioning in parallel for teachers and students, and, more ambitiously, the organization of math modelling summer camps. These would all be opportunities not only to learn about modelling but also, and more importantly, to live authentic modelling experiences.

In-class support from experts was also mentioned as a powerful way to support teachers in their integration of modelling in the classroom.

Dissemination of a classroom modelling culture could also benefit from the development of a bank of video examples of in-class modelling activities, where teachers could witness the implementation by their peers of strategies for dealing with uncertainty, and for “helping progress without killing the process”.

These various approaches could all contribute to developing and interconnecting modelling communities of practice.

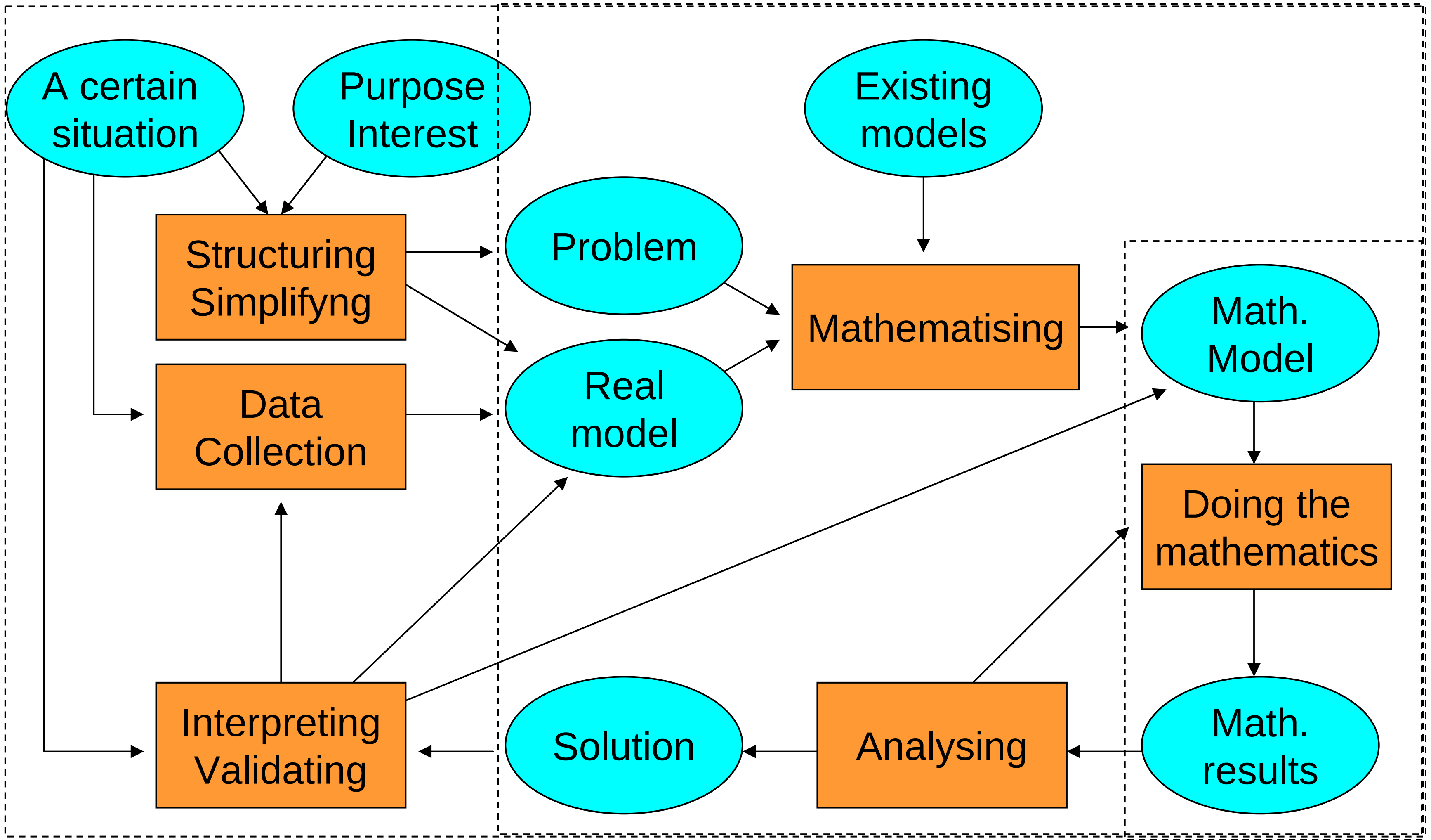
Mathematical Modelling and Science

A look at situations for modelling

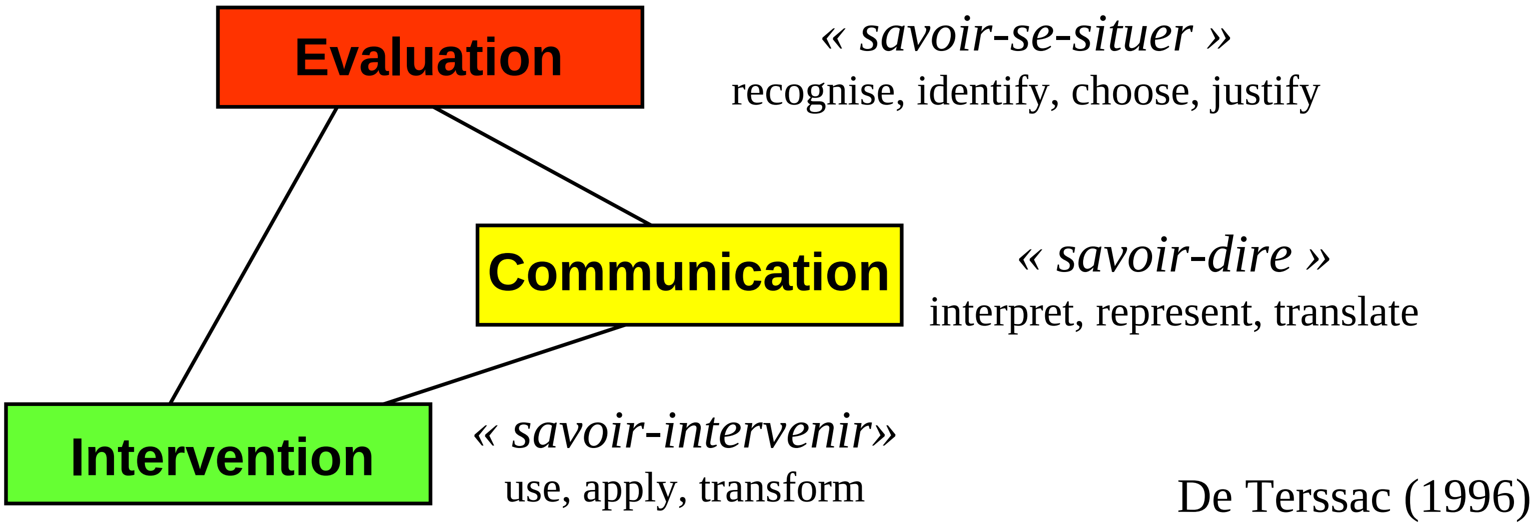
F. Caron, Université de Montréal
H. Squalli, Université de Sherbrooke
P. Etchecopar, J.P. Villeneuve, Cégep de Rimouski
M. Laforest, École Polytechnique de Montréal
J. Bélair, Université de Montréal

The modelling cycle

(from Blum *et al*, 2003)

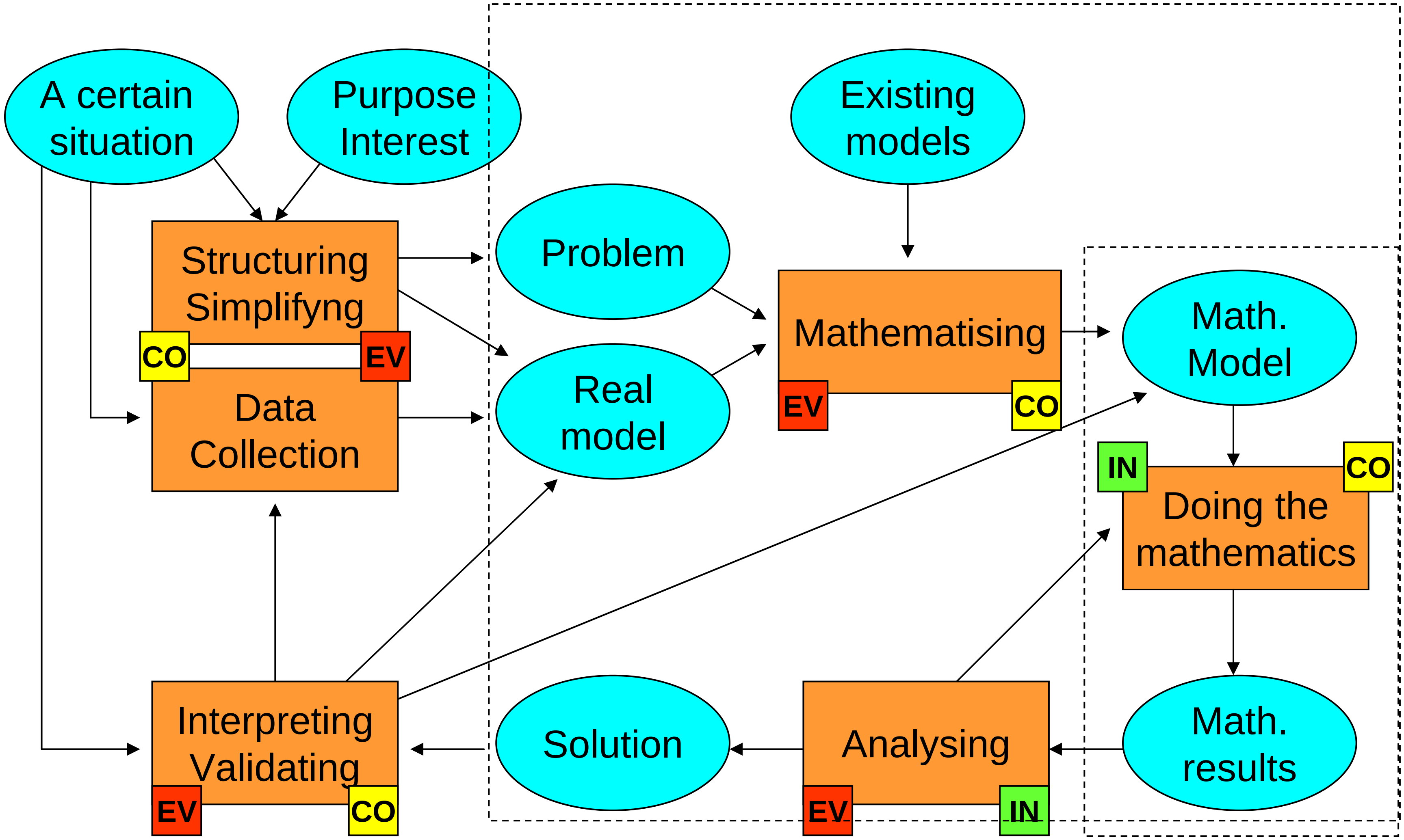


A typology of competencies

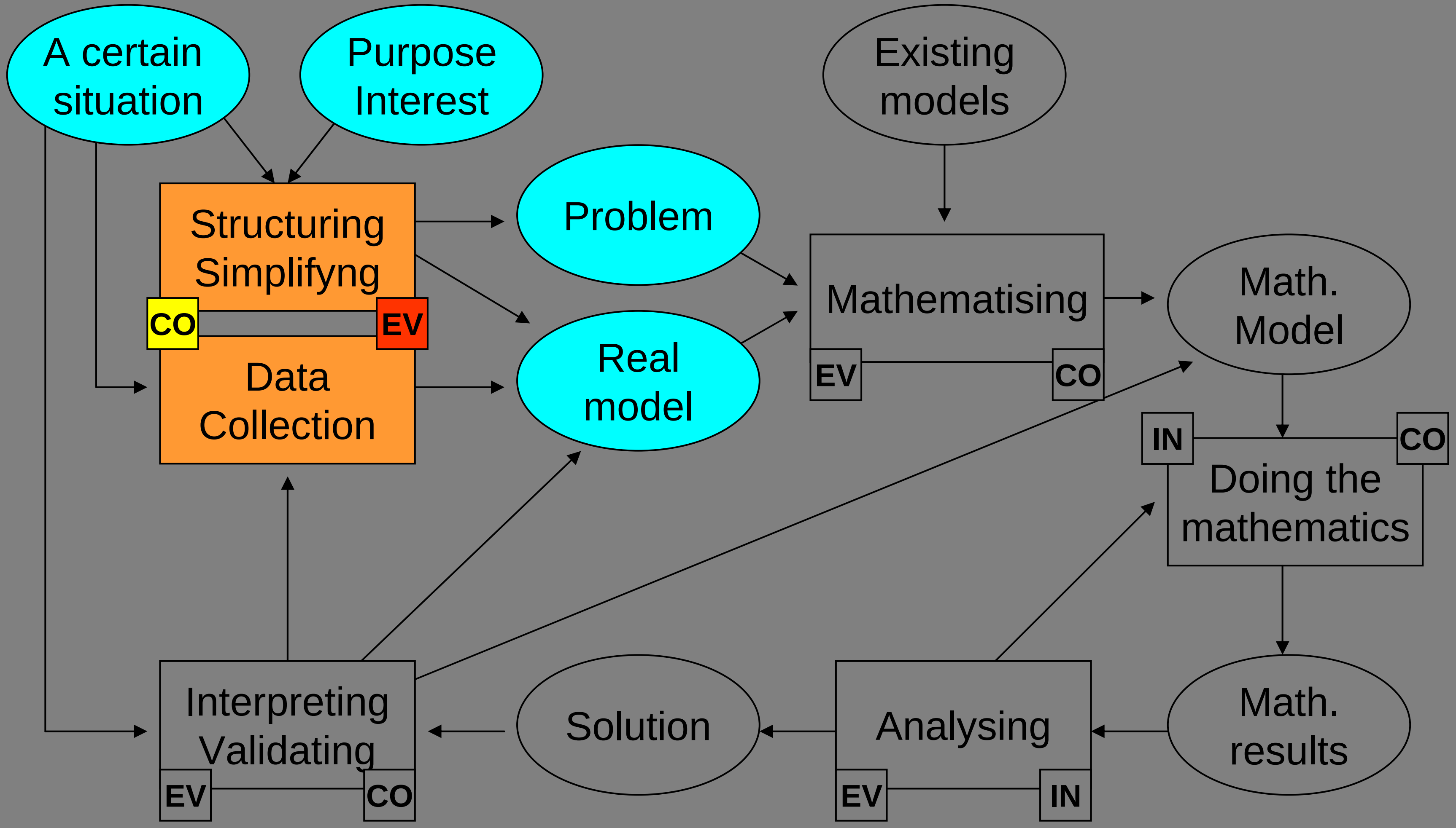


- These differents skills play an important role at specific stages of mathematical modelling.
- Modelling projects can be used to develop and assess competencies.

Modelling competencies

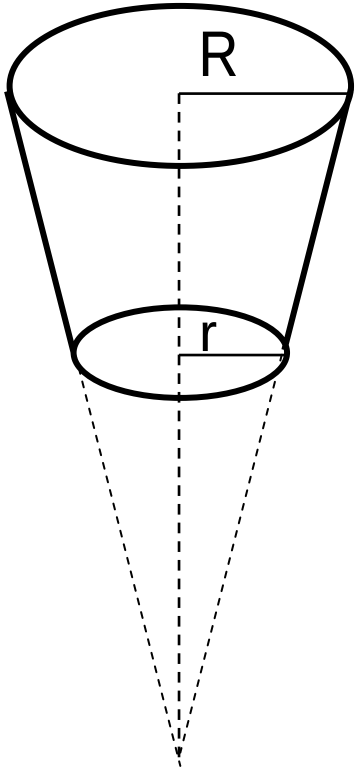


The modelling process



Defining the goal and structuring

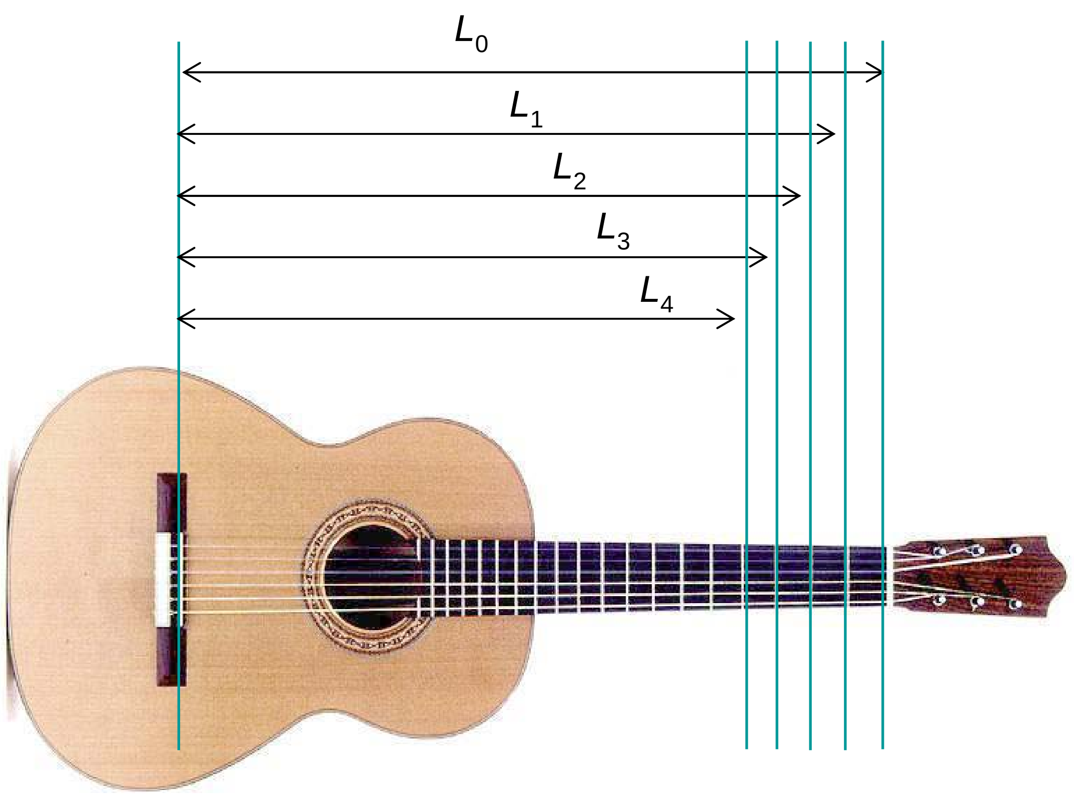
On cherche à faire un verre à café en carton qui contiendrait 250 ml.



Donnez les dimensions d'un tel verre et la surface de carton nécessaire à sa fabrication

Structuring and Data collection

Pourquoi les frettes d'une guitare sont-elles plus rapprochées lorsqu'on se dirige vers la caisse de résonance ?



n	Note	L_n (cm)
0	mi	65,5
1	fa	61,9
2	fa #	58,4
3	sol	55,1
4	sol #	52,1
5	la	49,2
6	la #	46,4
7	si	43,8
8	do	41,4
9	do #	39,1
10	ré	36,9
11	ré #	34,8
12	mi	32,8

CO EV

Structuring and Simplifying

- We are at the start of the 2008 U.S. presidential elections, and one important area of debate is sure to be the national debt.
- As high school students, you have a particular interest in this subject since you are the people who will pay off or at least manage the national debt in the future.
- *The rate at which the national debt changes depends on the difference between federal income (primarily taxes) and federal expenditures.*

Your first task is to build a model that can be used to help understand the national debt and make forecasts based on different assumptions.

As usual, modeling involves a balance between so much complexity that the model may be intractable and so little complexity that it is unrealistic and useless.

Your model needs, at the very least, to allow you to consider different tax policies and different expenditure policies

CO EV

Making sense of data

- As usual, raw numbers don't carry much information. Those numbers must be placed in context. For example, total national debt is less meaningful than national debt per capita.
- In addition, you must be careful about inflation. Many analysts look at the ratio between national debt and gross domestic product as a good indicator of the impact of the national debt.
- *Others worry about the cost of servicing the national debt. This cost is affected by both the size of the national debt and the interest rate the government must pay to borrow money.*

You may want to look at the Wikipedia article http://en.wikipedia.org/wiki/National_debt_by_U.S._presidential_terms for some figures involving the ratio between national debt and gross domestic product.

CO EV

Looking for variables and indicators

Une usine produit des balles de caoutchouc de différents diamètres et compositions.

On cherche à trouver un indicateur mathématique pour caractériser la qualité de rebondissement de ces balles.

CO EV

Measurable purpose

What is the impact of greenhouse gases on the Earth’s temperature?

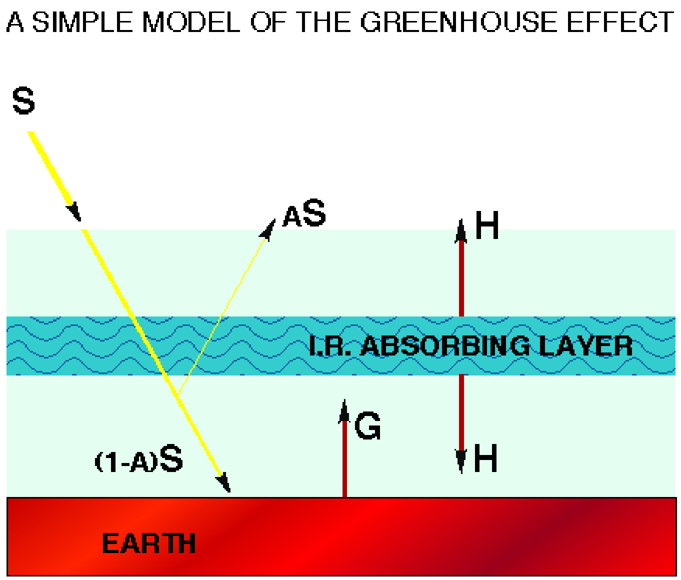
- Simplify : *Sun and Earth, both uniform*
- Structure : *Heat exchange problem*
- Reality : *Earth’s rotation & axis, elliptical orbit, complex atmospheric chemistry*
- Problem : *Relate quantities [GHS] and T*

CO EV

A real model

The effect of greenhouse gases is modeled by a thin absorbing layer.

- ➢Reduce dimension
- ➢Instantaneous effects
- ➢Preserve essential physics : energy balance



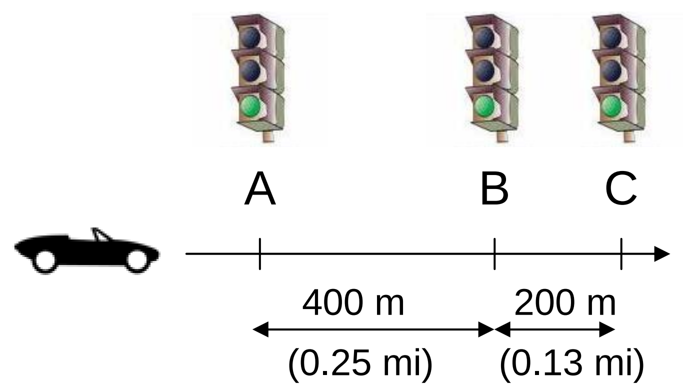
CO EV

Simplifying and making assumptions

- A stretch of a suburban road lined with shopping plazas carries heavy commuter traffic. The road has 15 traffic signals, unevenly spaced, at the intersections with cross streets and mall entrances.

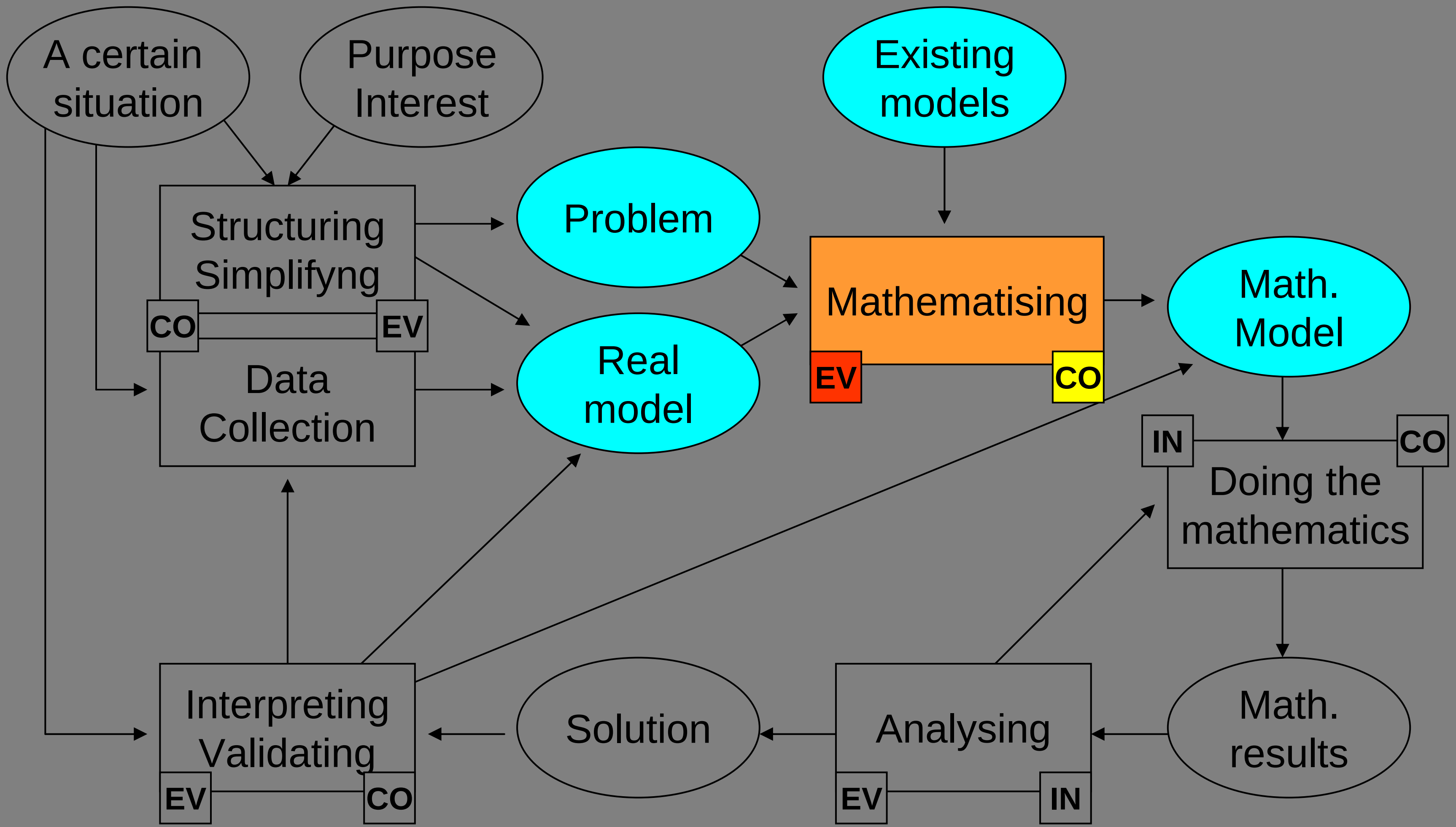
  How should we time the lights in order to maximize the flow of commuter traffic?

- Consider **first** a simpler case.
  - Suppose the road travels north-south and has only three lights A, B and C.
  - First consider only light A, and suppose a 1-minute cycle
    - green for 30 seconds
    - yellow for 5 seconds
    - red for 25 seconds.



High School Mathematics at Work. MSEB, 1998, 147-152

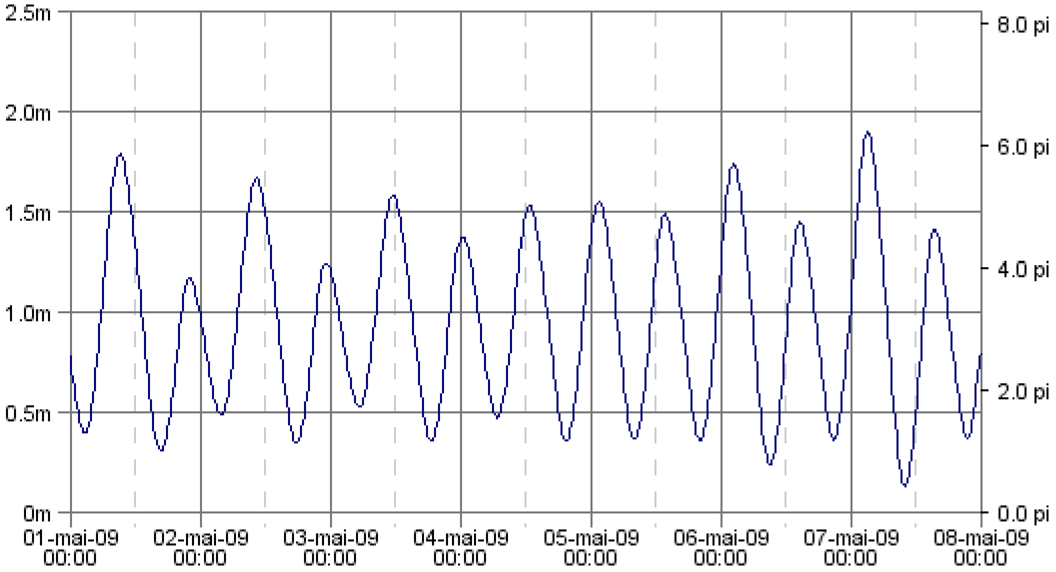
The modelling process



EV CO

Model? What model?

- Voici une prédiction des marées (heures et hauteurs des pleines et basses mers) de Bathurst au Nouveau Brunswick, pour le début de mai 2009 (www.marees.gc.ca) :



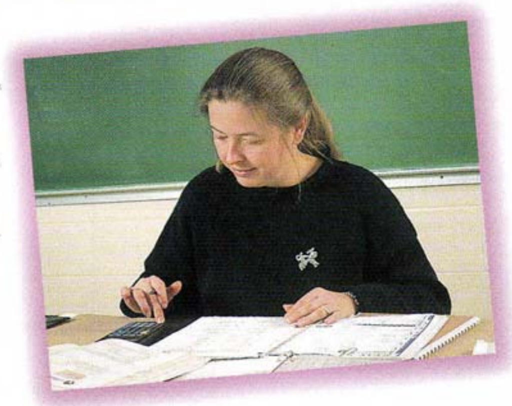
Comment pourrait-on utiliser ces « données » pour prédire les marées de la semaine suivante ?

The temptation of making reality fit the model

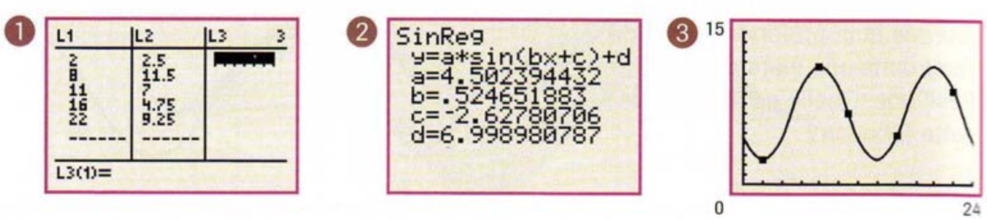
La **calculatrice** à affichage graphique permet de retrouver, par **régression,** la règle d'une fonction sinusoïdale à partir d'un ensemble de couples.

Une certaine journée, on a observé le niveau de la marée sur l'une des berges du golfe du Saint-Laurent en fonction de l'heure de la journée. La table de valeurs ci-contre montre quelques-uns de ces résultats.

Heure	Hauteur (en m)
2:00	2,50
8:00	
11:00	
16:00	
22:00	



h) Décris chacune des étapes de la démarche suivante qui permet d'établir, par régression, la règle d'une fonction sinusoïdale et d'en tracer le graphique.



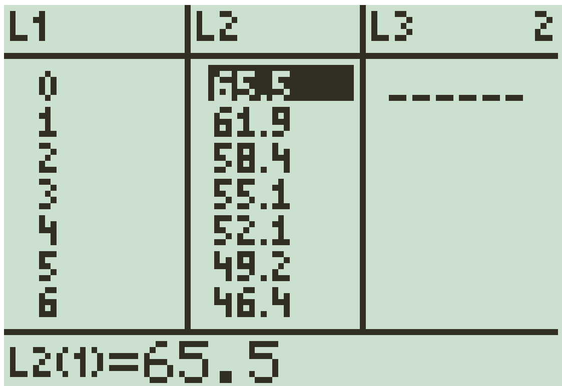
3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles

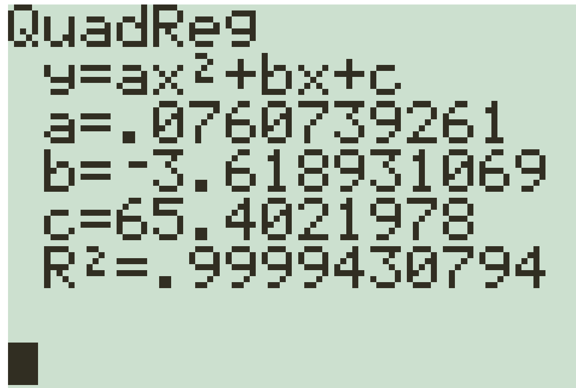
Sample problem: Investigate, using graphing technology in degree mode, and explain how the function $h(t) = 5\sin(30(t + 3))$ approximately models the relationship between the height and the time of day for a tide with an amplitude of 5 m, if high tide is at midnight.

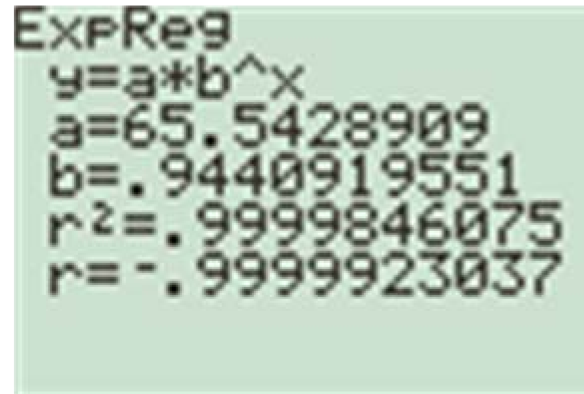
The Ontario Curriculum
Mathematics, Grades 11-12
Ministry of Education, 2007

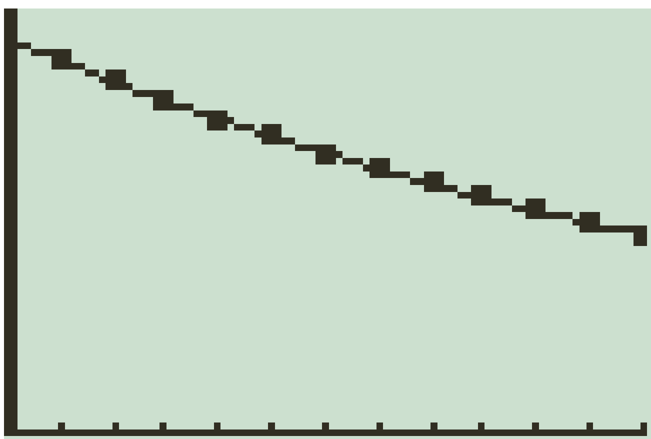
G. Breton *et al.*
Réflexions mathématiques 536
Les Éditions CEC, 1999

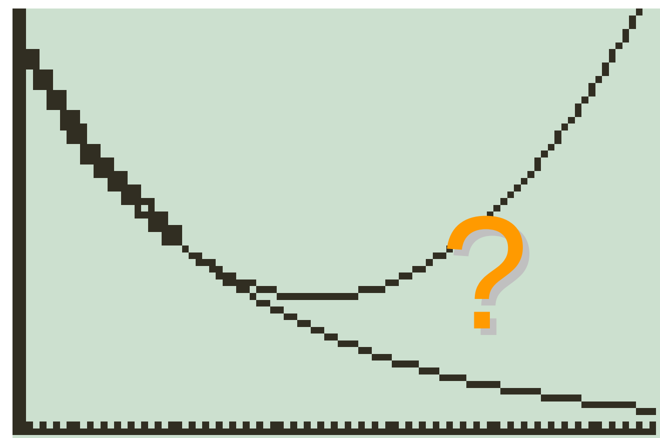
The limits of a good fit



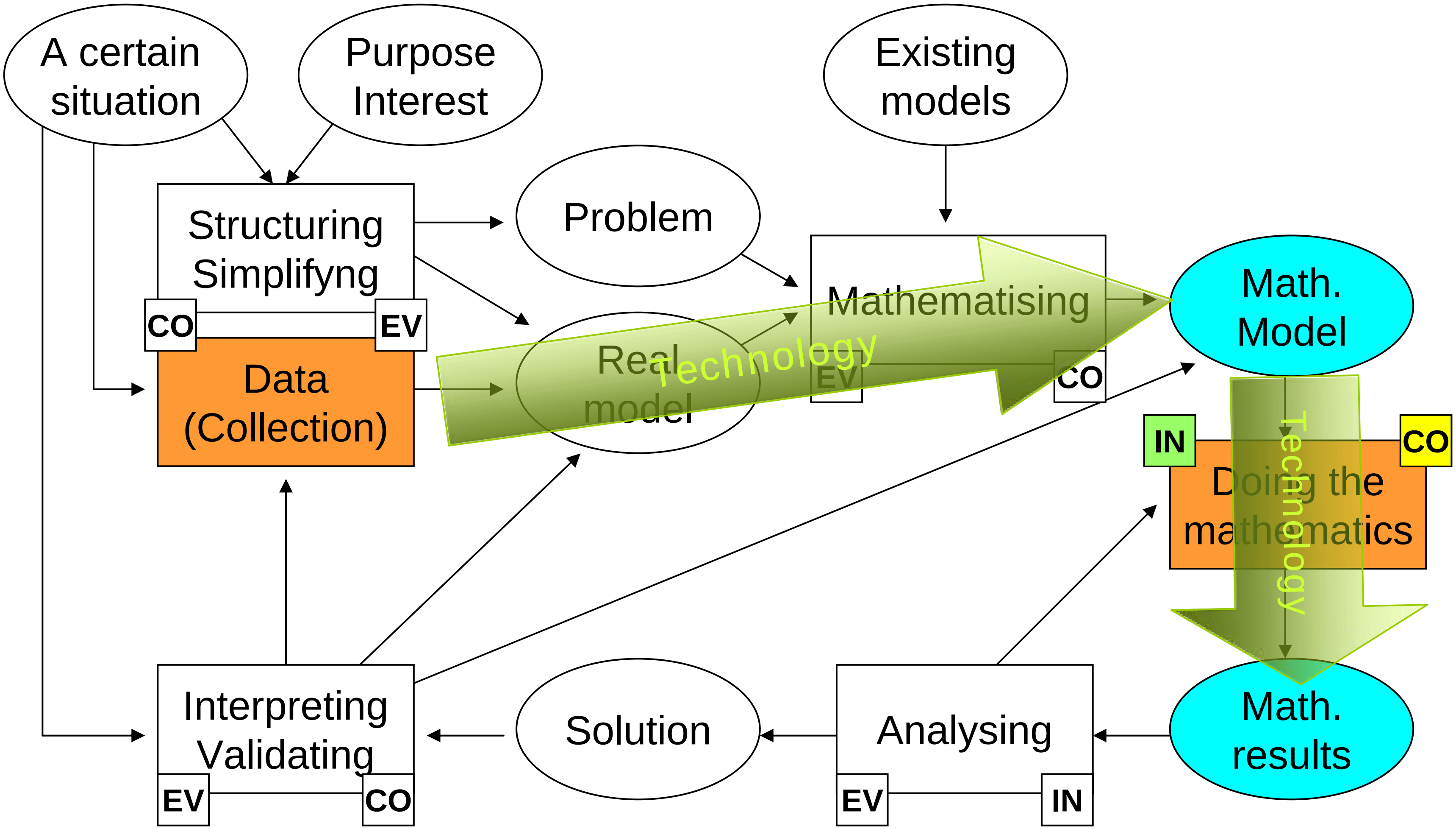








Bypassing the process



EV CO

Mathematatising

Quelle est la probabilité pour une femme canadienne de développer un cancer du sein au cours de sa vie ?

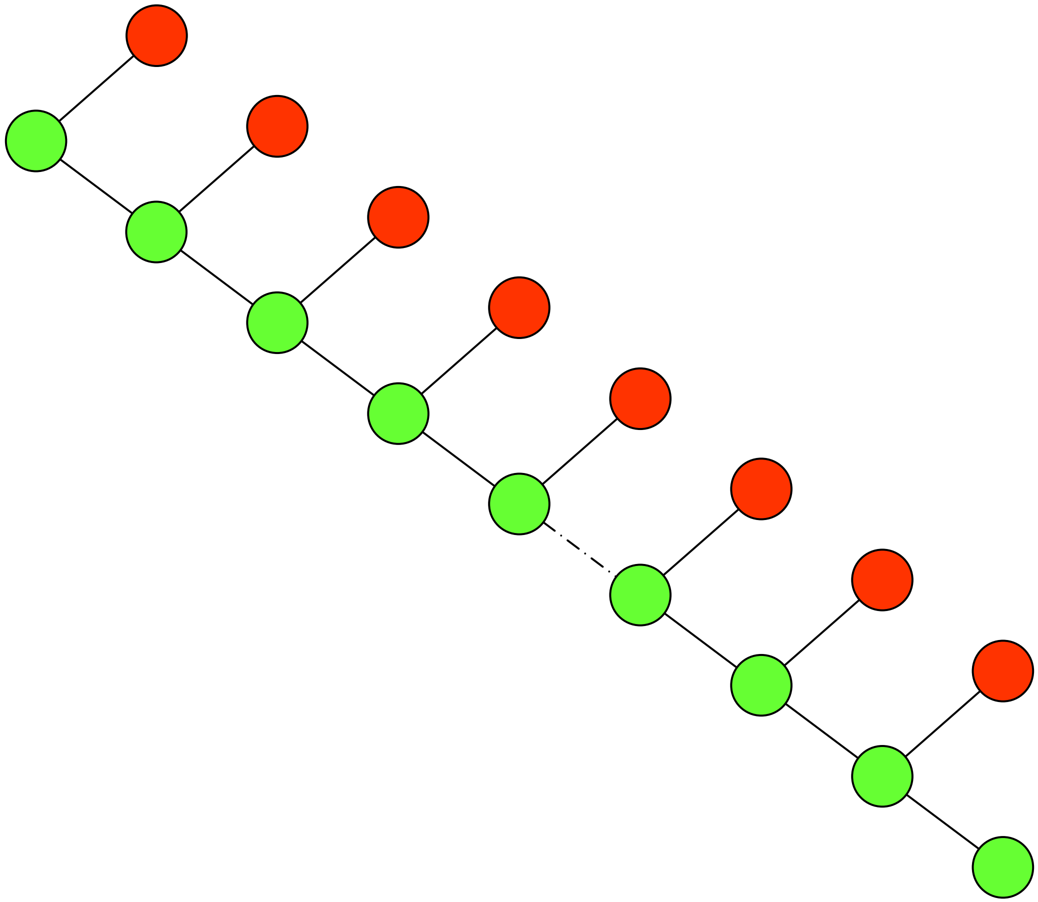
Groupe d’âge	Nombre de nouveaux cas au Canada selon l'âge	Nombre de décès au Canada selon l'âge
0 à 19 ans	5	-
20 à 29 ans	75	5
30 à 39 ans	840	100
40 à 49 ans	3 500	440
50 à 59 ans	6 100	940
60 à 69 ans	5 500	1 050
70 à 79 ans	3 700	1 100
80 ans et +	2 600	1 700

http://www.cancer.ca/
Statistiques canadiennes sur le cancer 2008

	Canada	Hommes	Femmes
Groupes d'âge	nombre de personnes (en milliers)		
Population totale	**32 976,0**	**16 332,3**	**16 643,7**
Moins de 5 ans	**1 740,2**	890,7	849,5
5 à 9 ans	**1 812,4**	927,2	885,2
10 à 14 ans	**2 060,5**	1 057,1	1 003,4
15 à 19 ans	**2 197,7**	1 126,2	1 071,6
20 à 24 ans	**2 271,6**	1 161,8	1 109,9
25 à 29 ans	**2 273,3**	1 148,5	1 124,7
30 à 34 ans	**2 242,0**	1 129,6	1 112,5
35 à 39 ans	**2 354,6**	1 185,1	1 169,5
40 à 44 ans	**2 640,1**	1 326,4	1 313,7
45 à 49 ans	**2 711,6**	1 356,4	1 355,2
50 à 54 ans	**2 441,3**	1 209,6	1 231,7
55 à 59 ans	**2 108,8**	1 040,5	1 068,3
60 à 64 ans	**1 698,6**	834,9	863,7
65 à 69 ans	**1 274,6**	614,5	660,1
70 à 74 ans	**1 047,9**	492,2	555,7
75 à 79 ans	**894,7**	398,6	496,1
80 à 84 ans	**650,8**	257,6	393,2
85 à 89 ans	**369,3**	125,5	243,7
90 ans et plus	**186,2**	49,9	136,3

Statistique Canada - http://www40.statcan.ca/l02/cst01/demo10a_f.htm

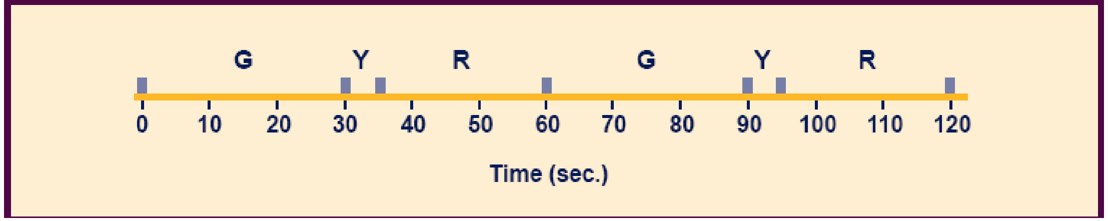
Diagrams as models



EV CO

Graphs as models

FIGURE 1: The green-yellow-red cycle of light A



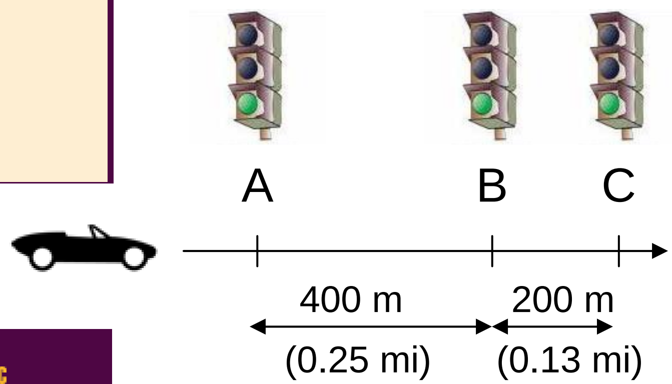
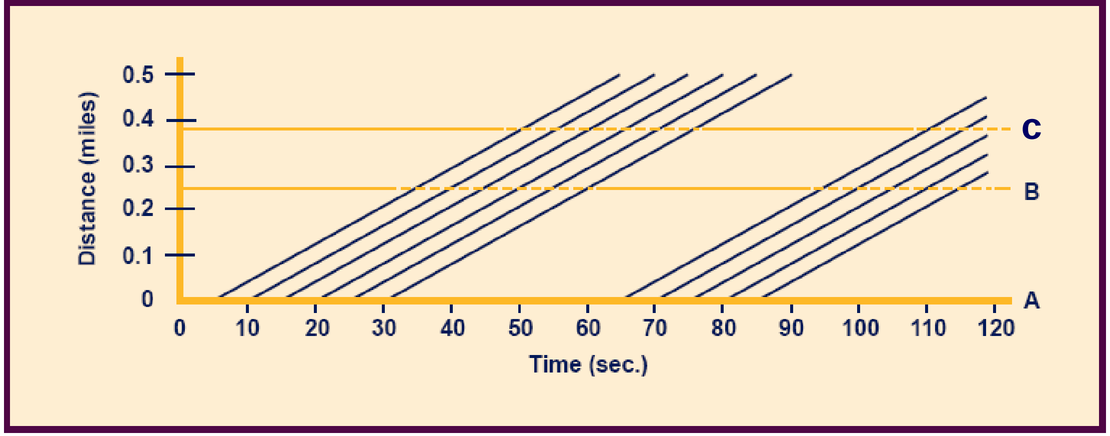


FIGURE 3: Determining when lights B and C should be green for north-bound traffic



High School Mathematics at Work. MSEB, 1998, 147-152

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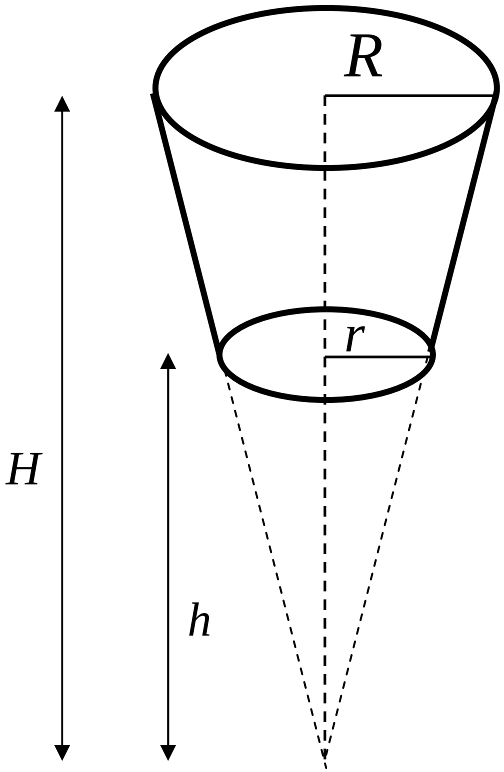
Looking for geometric relations

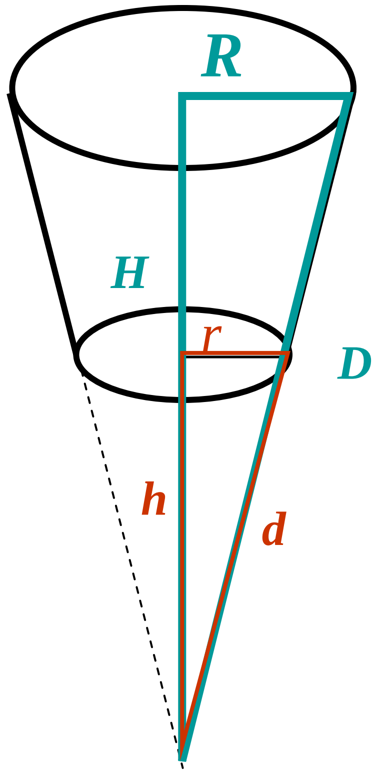


$$\frac{\pi \cdot R^2 H}{3} - \frac{\pi \cdot r^2 h}{3} = 250$$

$$\frac{r}{R} = \frac{h}{H} = k$$

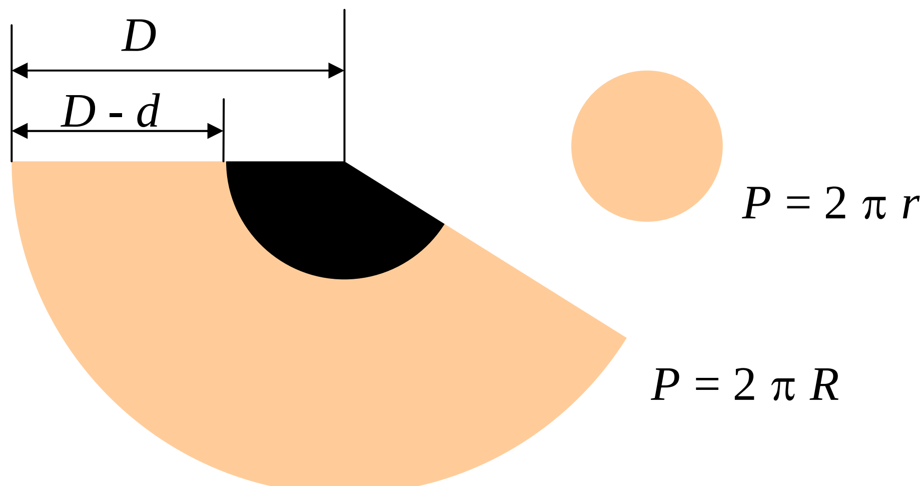
$$\pi \cdot R^2 H \cdot (1 - k^3) = 750$$

Looking for geometric relations



$$D^2 = R^2 + H^2$$

$$d^2 = r^2 + h^2$$



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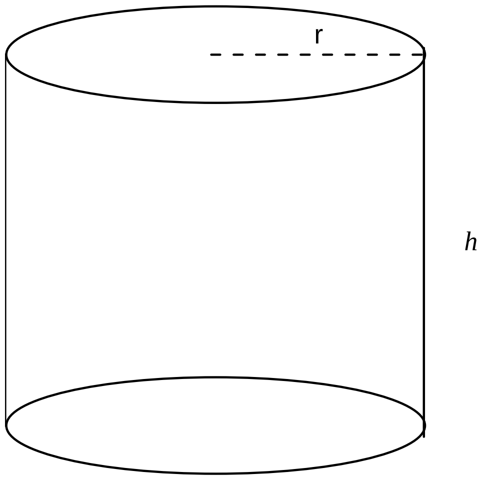
Classical Problems

- Find the dimensions of a cylindrical tank to minimize the surface



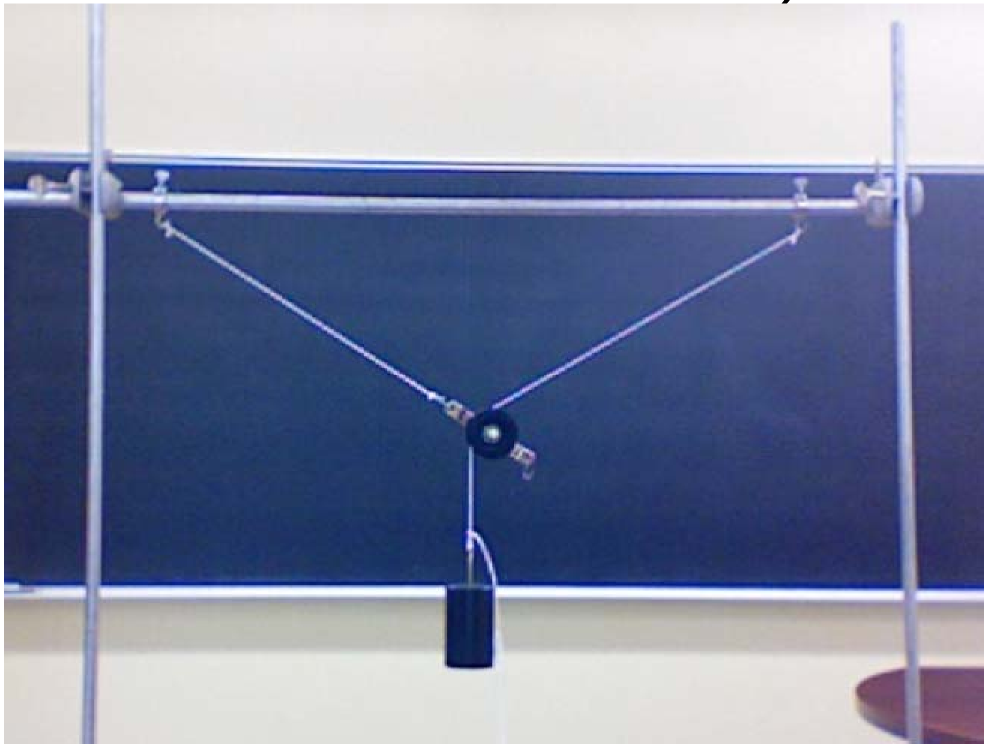
- Mathematatising

$$S = 2\pi r^2 + 2\frac{V}{r}$$

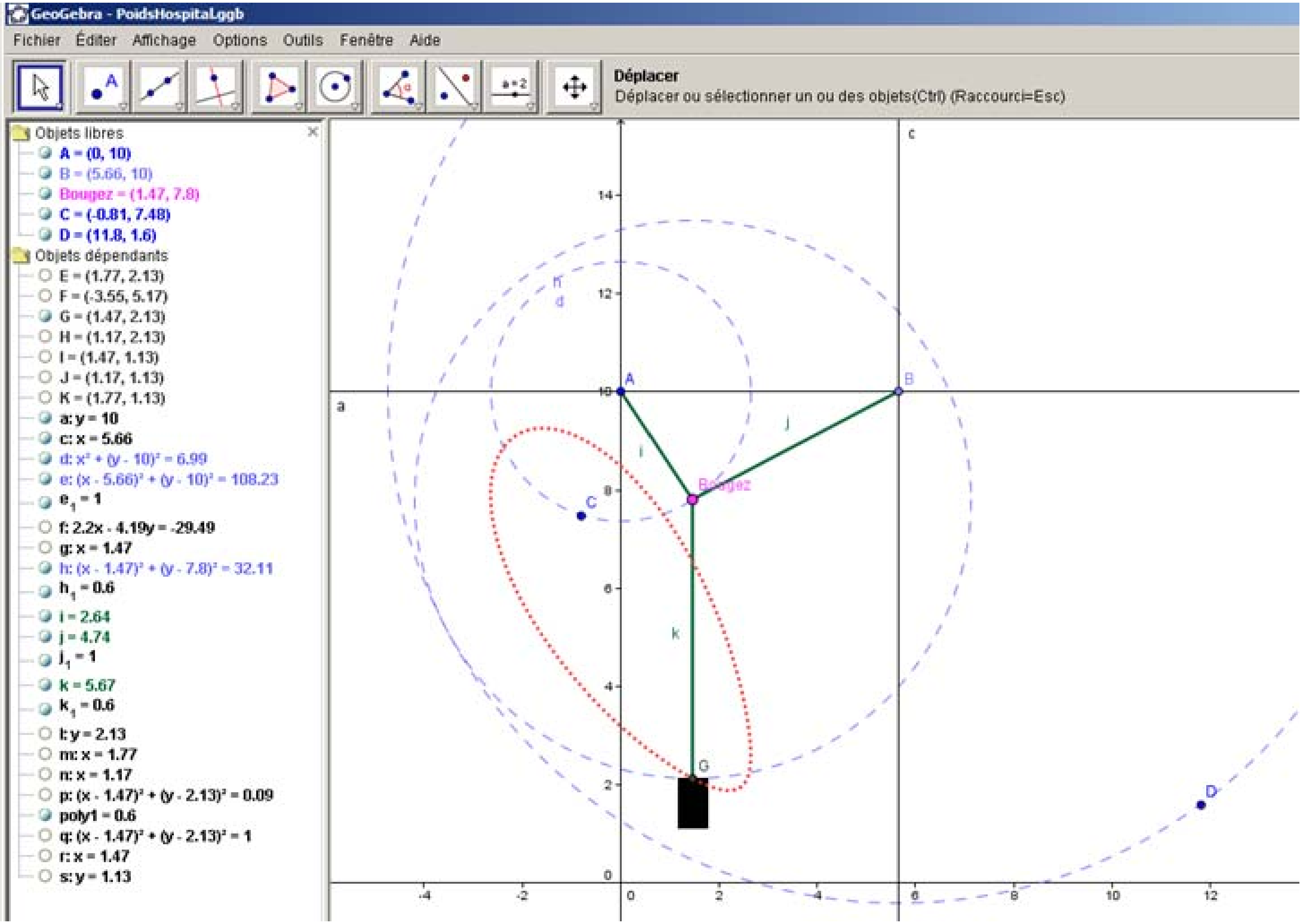
$$\frac{dS}{dr} = 0 \quad \text{et} \quad \frac{d^2S}{dr^2} > 0$$

Building a dynamic geometry model

- Le poids de L’Hospital
  (Partenariat ÉTS/CEGEP/SECONDAIRE 2009)



Building a dynamic geometry model



EV CO

Looking for patterns in data

$f = k / L$

n	Note	L_n (cm)	$L_n - L_{n-1}$	L_n / L_{n-1}	L_n / L_0
0	mi	65,5			**1,0000**
1	fa	61,9	-3,6	**0,9450**	0,9450
2	fa #	58,4	-3,5	**0,9435**	0,8916
3	sol	55,1	-3,3	**0,9435**	0,8412
4	sol #	52,1	-3	**0,9456**	0,7954
5	la	49,2	-2,9	**0,9443**	**0,7511**
6	la #	46,4	-2,8	**0,9431**	0,7084
7	si	43,8	-2,6	**0,9440**	**0,6687**
8	do	41,4	-2,4	**0,9452**	0,6321
9	do #	39,1	-2,3	**0,9444**	0,5969
10	ré	36,9	-2,2	**0,9437**	0,5634
11	ré #	34,8	-2,1	**0,9431**	0,5313
12	mi	32,8	-2	**0,9425**	**0,5008**

EV CO

Using recurrence relations

Durée de vie d'un site d'enfouissement

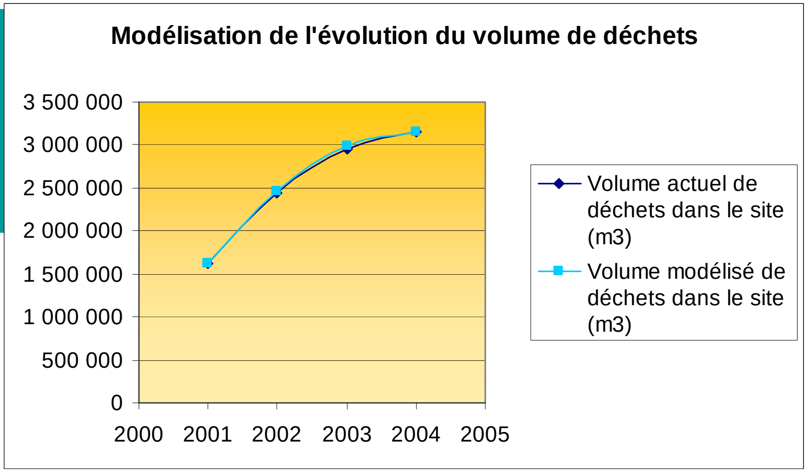
V_n = **Le volume de déchets au début de l'année *n***

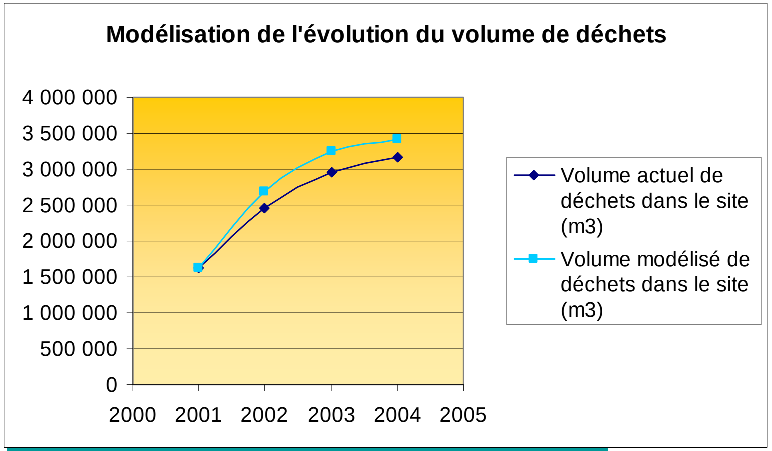
Q_n = **La quantité de déchets acheminée à l'année *n***

Modèle 2 :

$$V_n = k\,(V_{n\text{-}1} + Q_{n\text{-}1})$$

$$k = 0{,}92$$



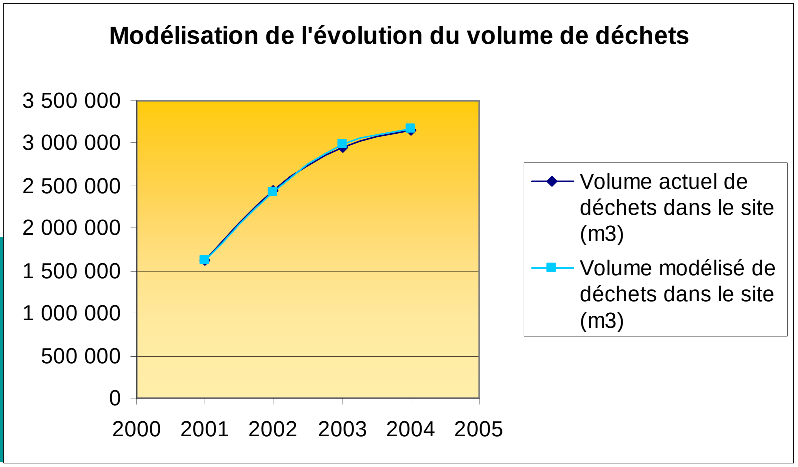


Modèle 1 : $V_n = V_{n\text{-}1} + Q_{n\text{-}1}$

Modèle 3 :

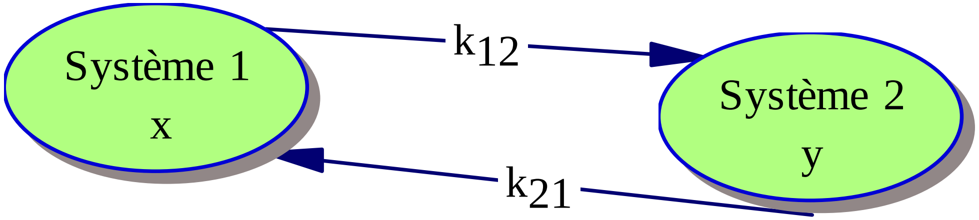
$$V_n = k_1\,V_{n\text{-}1} + k_2\,Q_{n\text{-}1}$$

$$k_1 = 0{,}94 \qquad k_2 = 0{,}85$$



Mathematising with Existing Models

- General case for Integral Calculus (Environnement)
  - Schéma compartimental



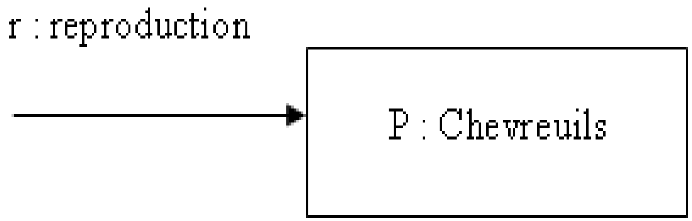
  - Modèle mathématique existant

$$\frac{dx}{dt} = k_{2,1}y - k_{1,2}x$$

$$\frac{dy}{dt} = k_{1,2}x - k_{2,1}y$$

Verhulst Model

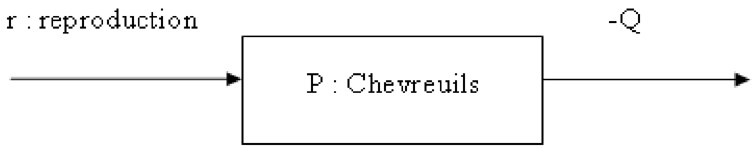
- Évolution du troupeau sans prélèvement



- Mathematising

$$P' = rP\left(1 - \frac{P}{K}\right)$$

- Évolution du troupeau avec prélèvement



- Mathematising

$$P' = rP\left(1 - \frac{P}{K}\right) - Q$$

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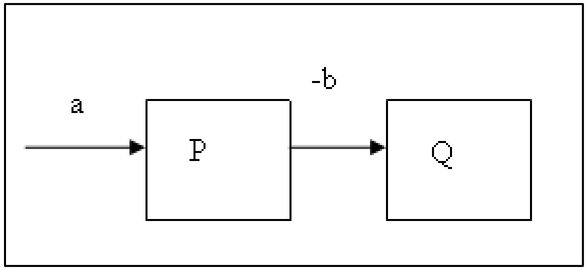
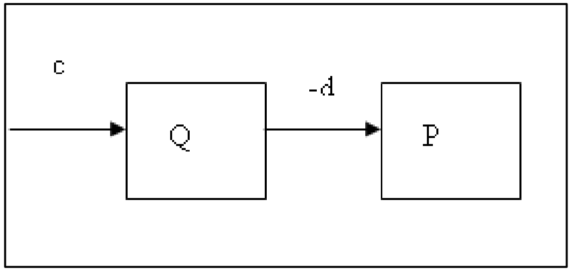
Populations in competition

- Deer (P) and Bear (Q): two species that fight for the same food





- Mathematising

a and c: Increasing Rate

b and d: decreasing Rate

$$P' = aP - bQ$$

$$Q' = cQ - dP$$

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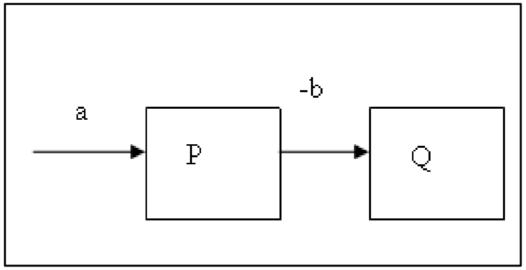
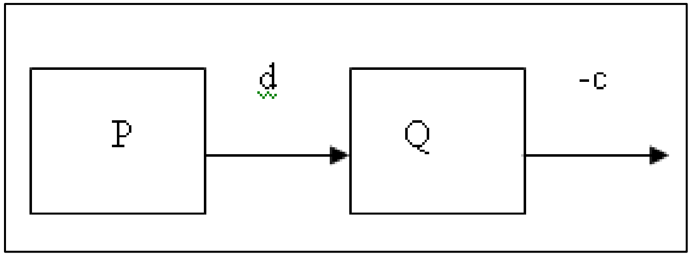
Volterra Model

- Predators and Preys (Sardine (P) and Shark (Q)): a species that eats another species





- Mathematatising

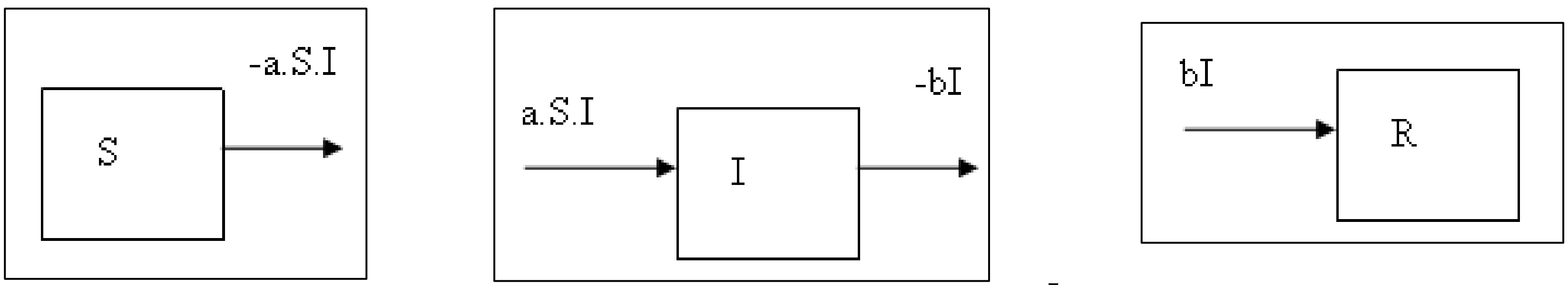
$$P' = aP - bPQ$$

$$Q' = -cQ + dPQ$$

a and c: Increasing Rate
b and d: Meeting Probability

Epidemy propagation (SIR Model)

- Healthy population (S), Sick Population (I) and Immune Population(R)



- Mathematising

  a: Infection Rate
  b: Recovery Rate

$$\frac{dS}{dt} = -aSI$$

$$\frac{dI}{dt} = aSI - bI$$

$$\frac{dR}{dt} = bI$$

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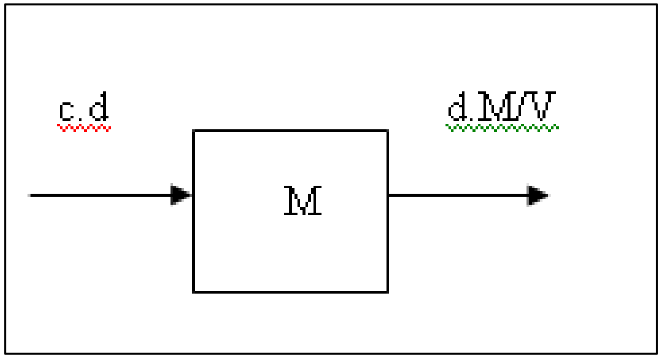
Lake Pollution

- A toxic substance is constantly dumped into a lake



- Mathematising

  d: Flow Rate

  c: Concentration

  V: Volume of the lake

$$M' = d.c - d.\frac{M}{V}$$

Existing models (physics)

Stephan-Boltzmann law A black body (non-reflecting) at temperature T emits a heat flux

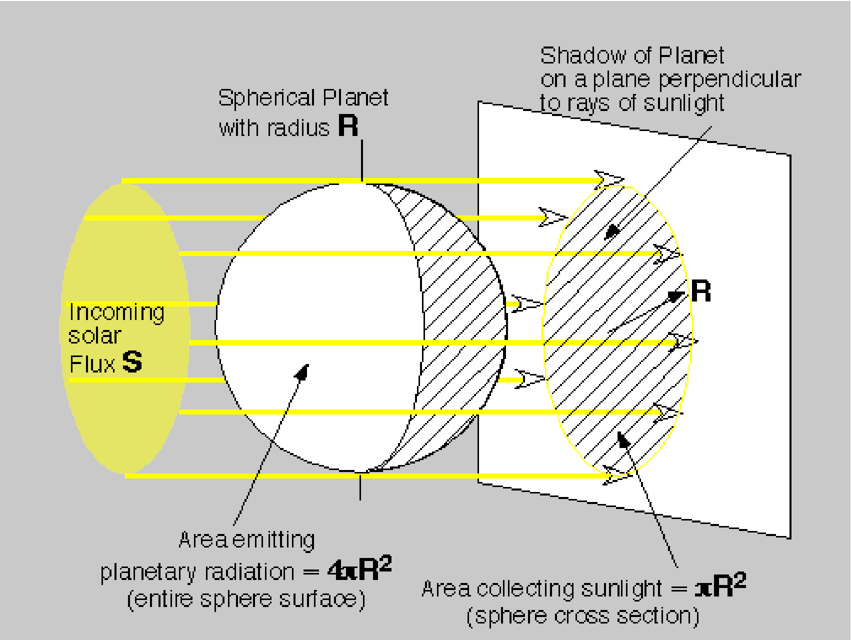
$$F = \sigma T^4$$

Geometry, proportionality, physics

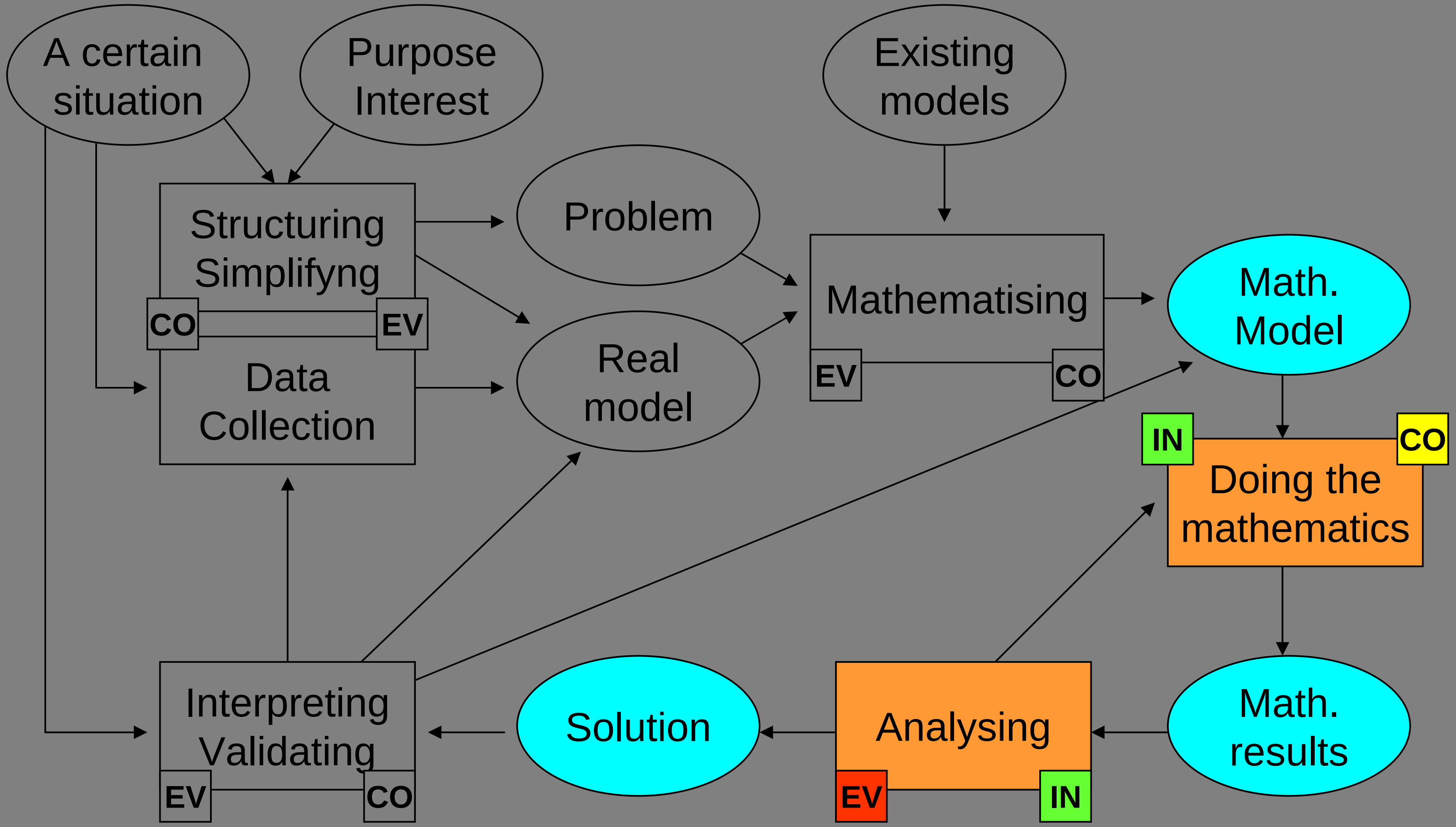
$$S = (1-a)F_S\left(\frac{R_S}{D}\right)^2$$

$$S = F_E 4\pi R_E^2$$

A Spherical Planet Receiving the Sun's Radiation



The modelling process



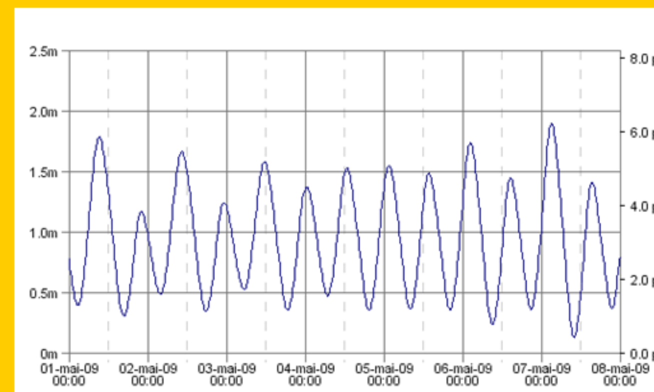
Using technology for tackling complexity

$f_1(x) = a_1 \sin b_1 (x-h_1) + k_1$

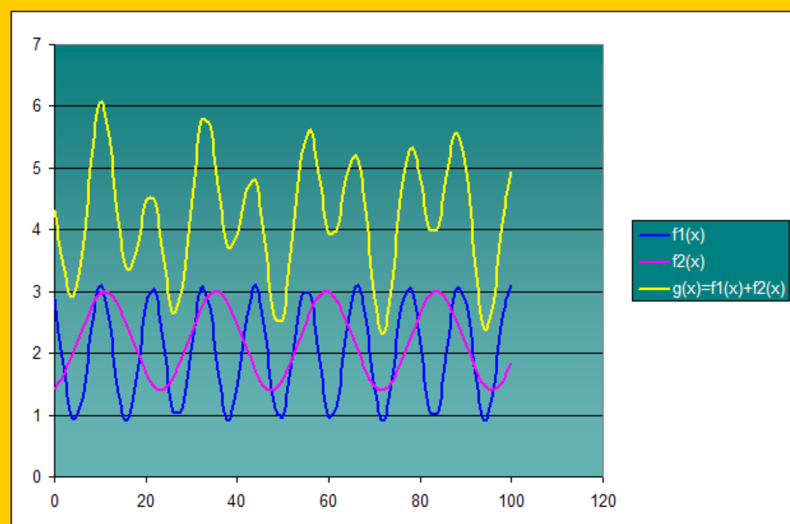
a_1	1,1
b_1	0,56
h_1	7,3
k_1	2,0

$f_2(x) = a_2 \sin b_2 (x-h_2) + k_2$

a_2	0,8
b_2	0,26
h_2	5,1
k_2	2,2

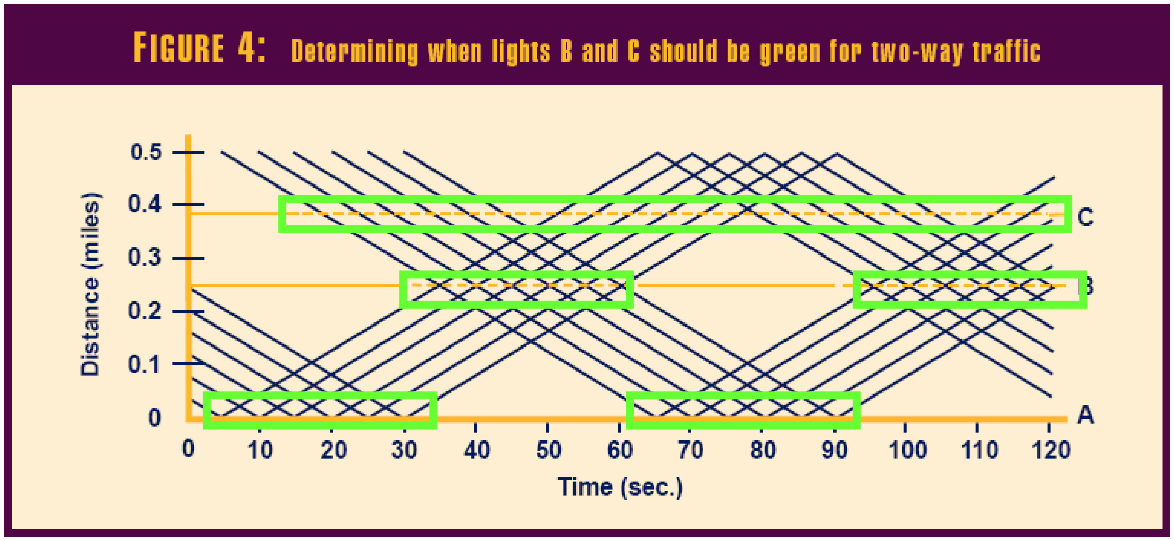


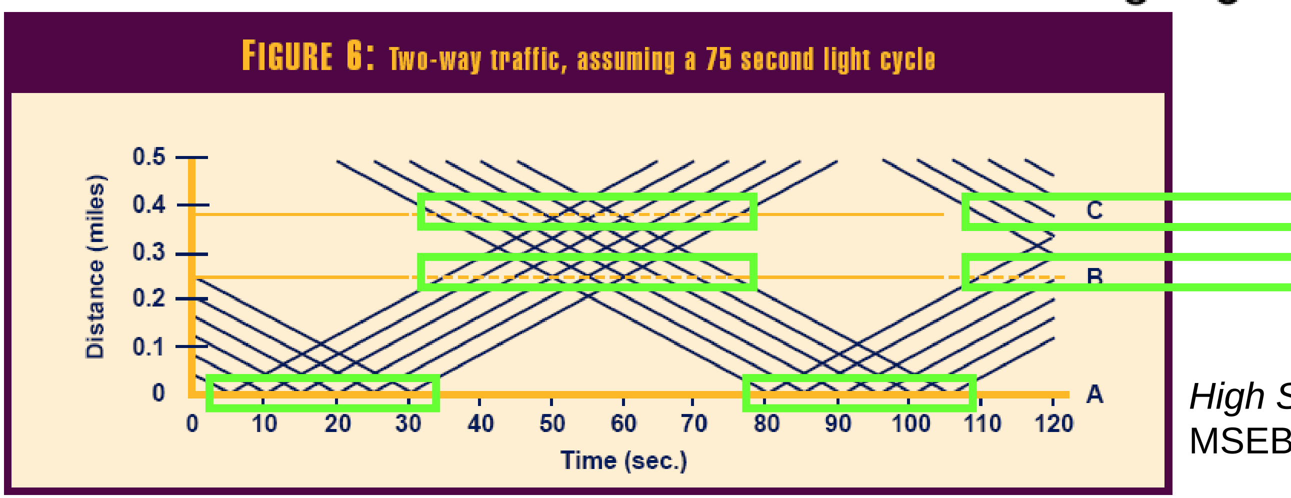
x	$f_1(x)$	$f_2(x)$	$g(x)=f_1(x)+f_2(x)$
0	2,892452521	1,42385063	4,316303156
2	1,810005674	1,62278129	3,432786959
4	0,941993093	1,97430642	2,916299516
6	1,268084251	2,38549628	3,653580535
8	2,420241219	2,74764804	5,167889262
10	3,098099193	2,96502267	6,063121861
12	2,536603867	2,98015464	5,516758512
14	1,369478577	2,78904366	4,158522237
16	0,913981902	2,44221213	3,356194028
18	1,684203379	2,031349	3,715552377
20	2,81084401	1,66507072	4,475914735
22	3,022337624	1,44020723	4,462544857
24	2,079985105	1,4162038	3,496188908
26	1,047358588	1,59940602	2,64676461
28	1,089916614	1,94138225	3,031298864
30	2,1596267	2,35172709	4,511353791
32	3,049176489	2,72196111	5,771137598
34	2,754588858	2,95420863	5,708797491
36	1,608345751	2,98707232	4,595418074
38	0,904137379	2,81186428	3,716001663
40	1,436758034	2,4749028	3,911660833
42	2,605073346	2,06526756	4,670340905
44	3,090481637	1,69125041	4,781732045
46	2,345134068	1,45172713	3,796861202
48	1,210256073	1,4100185	2,620274575
50	0,9667108	1,57715068	2,543861475
52	1,889371994	1,90894032	3,798312312
54	2,936891839	2,31767498	5,254566815
56	2,927002663	2,69530088	5,622303547

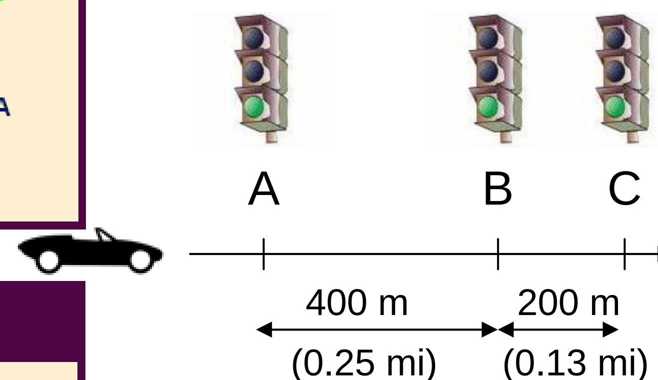


IN CO

Solving graphically by playing with parameters







High School Mathematics at Work. MSEB, 1998, 147-152

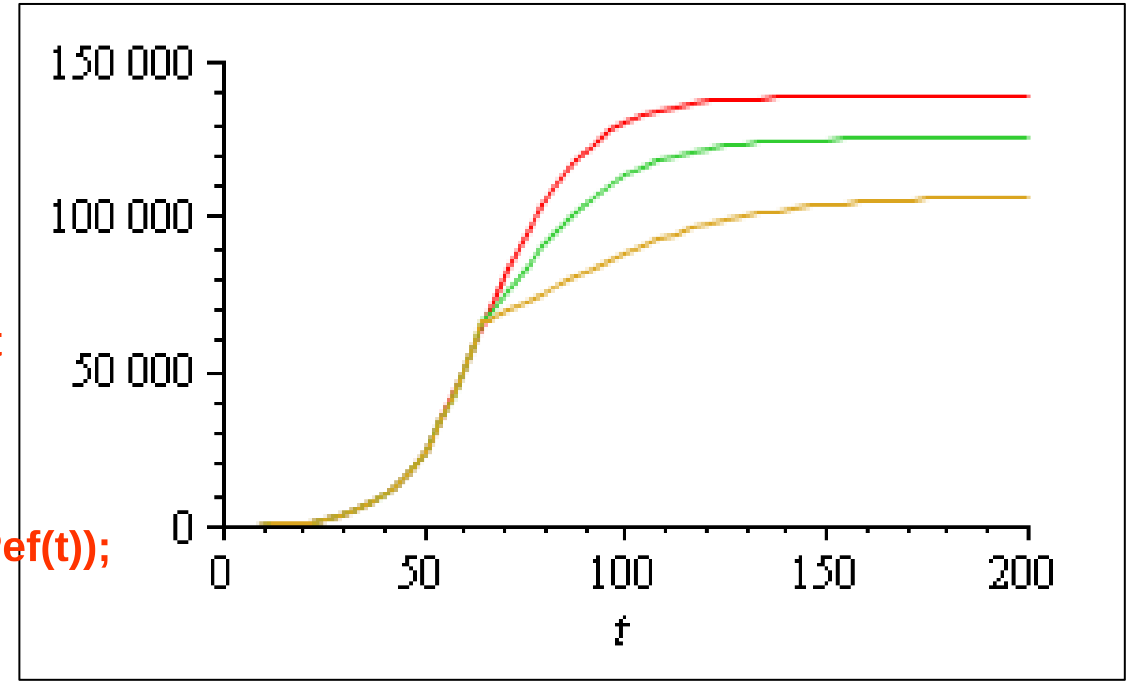
IN CO

Parameter Simulations

- Verhulst Model
  - Simulations of the Hunting Quota (Q)

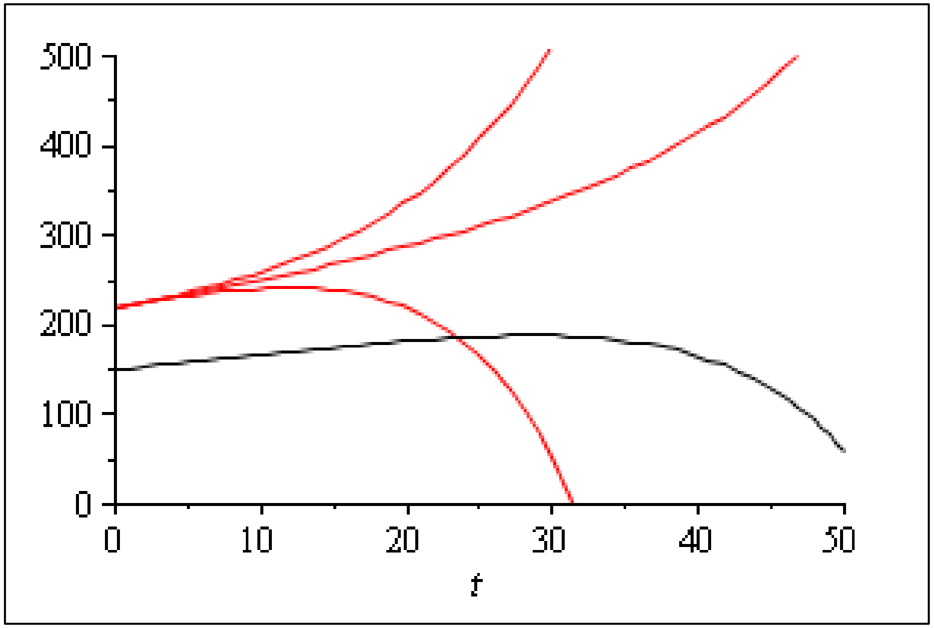
$$P' = rP\left(1-\frac{P}{K}\right)-Q$$

```
>restart;
(…)
K:=150000;r:=0.098;
#Prélèvement
ListeQ:=[1000,2000,3000];
#Début des prélèvements
M:=64;
#La population avec prélèvement
ci2:=P(M)=Nef(M);
eqd2:=P1=r*P(t)*(K-P(t))/K-Q;
(…)
f:=t->piecewise(t<0,0,t<M,Nef(t),Pef(t));
listef:=[seq(f(t),Q=ListeQ)];
plot(listef,t=0..200,0..K);
```



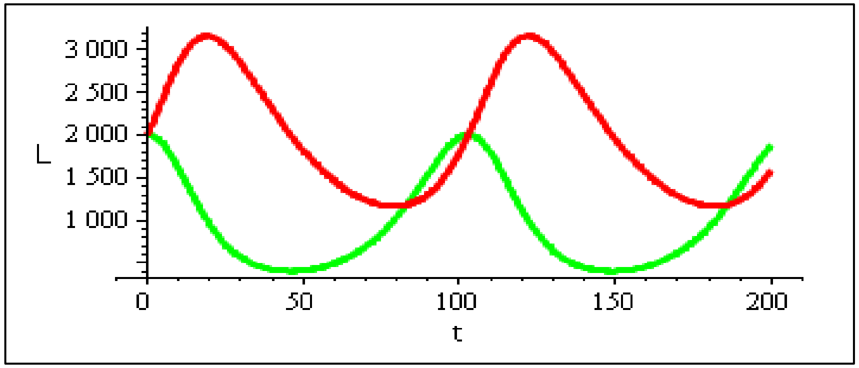
Parameter Simulations

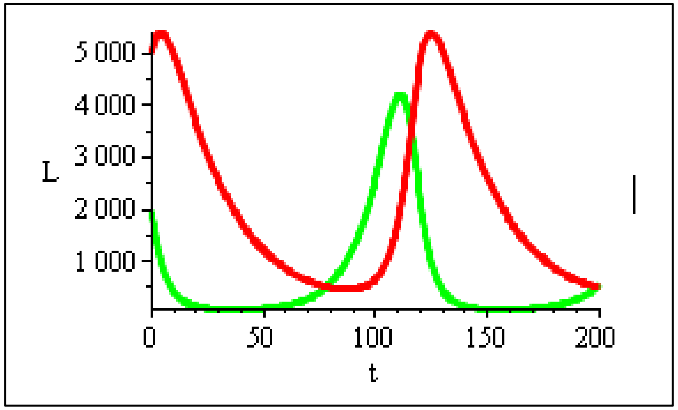
- Population in competition
  - Simulations of the increasing rate of one population



Parameter Simulations

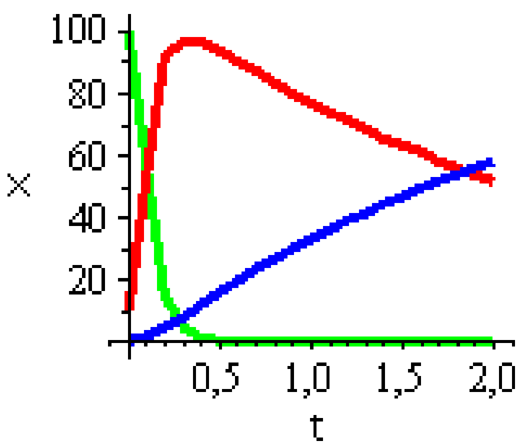
- Volterra Model (Predator and Prey)
  - Simulations of the initial conditions of predators

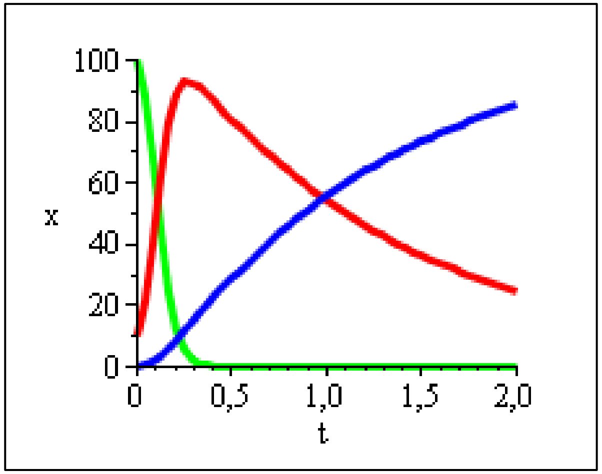




Parameter Simulations

- Volterra Model
  - Simulations of the recovery factor





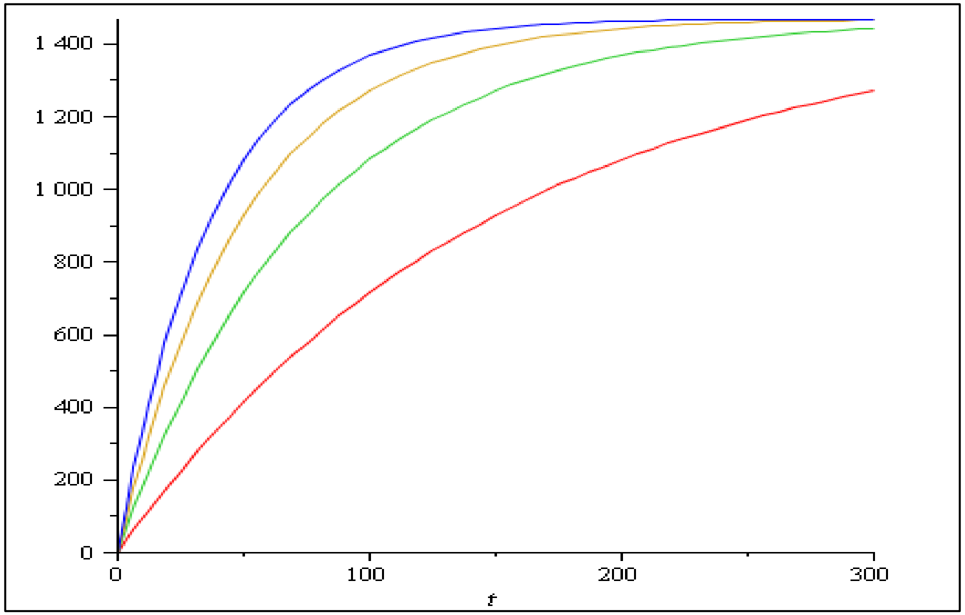
Healthy population

Sick Population

Recovered Population

Parameter Simulations

- Lake Pollution
  - Simulations of the flow rate



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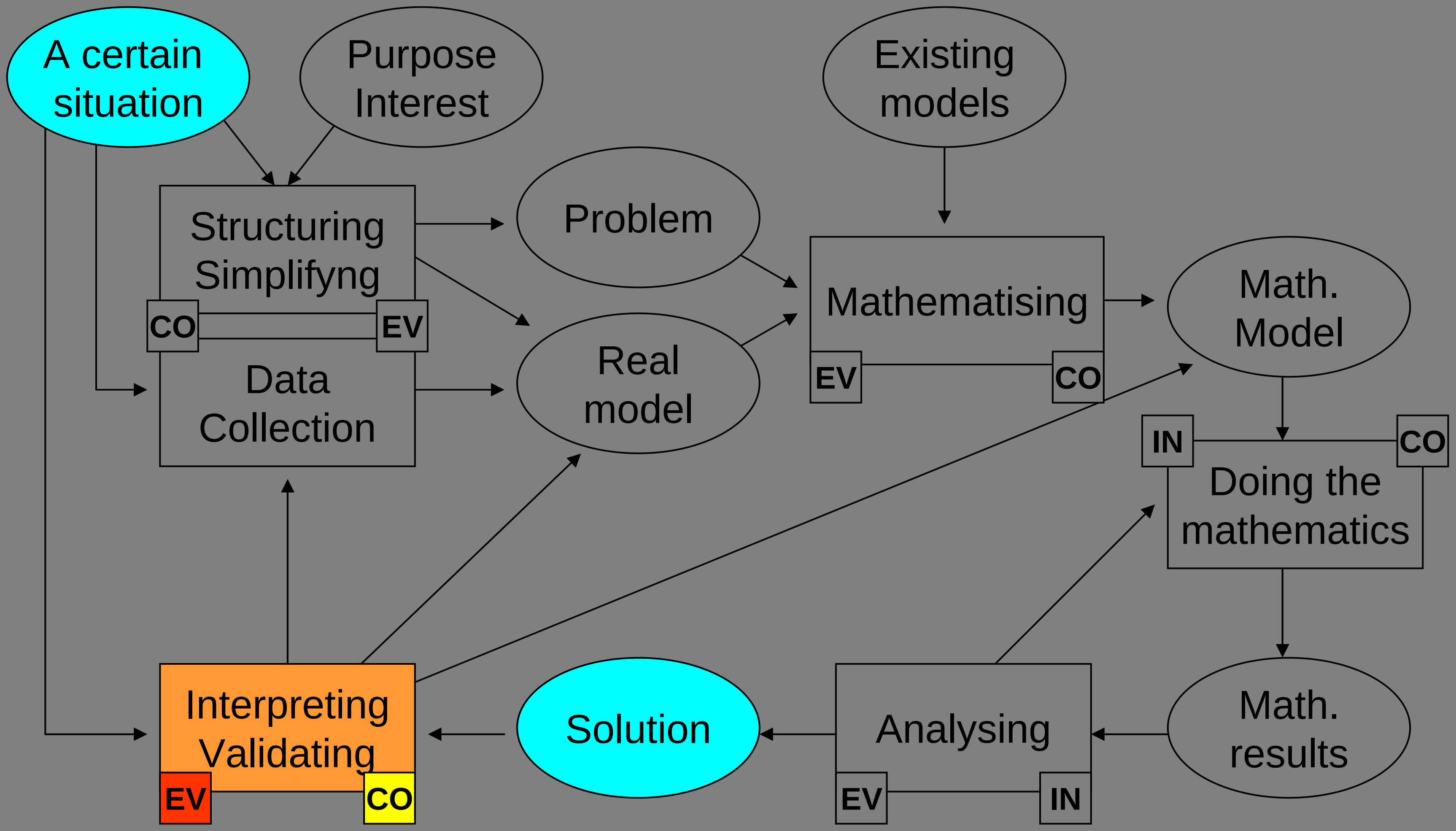

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Last Step in Modelling

- Summary of the results
  - Give a complete answer to the problem
  - Write the final report, including the Maple standard print out.
- Critical examination of the model
  - Find some limitations of the mathematical model used to solve the problem

Cégep de Rimouski

EV CO

Validate model

$$T_P = T_S \sqrt[4]{2(1-a)} \sqrt{\frac{R_S}{2D}}$$

Known quantities : temperature of Sun Ts, albedo a,
radius of Sun Rs, distance
between Sun and Earth D

Predicted temperature : Tp = 25 C
Measured temperature : Tp = 15 C

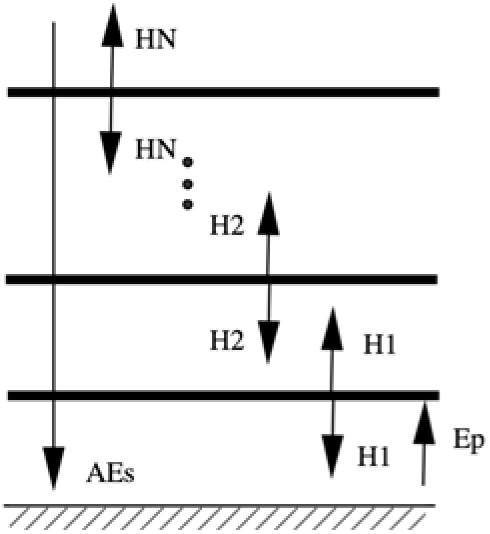
EV CO

Improve model

- ➢Variable thickness of layer of greenhouse gases
- ➢Melting snow and impact on albedo
- ➢Water vapour (a GHG) and temperature
- ➢Variable distance Sun-Earth
- ➢…

N layer model

$$T_P = T_S \sqrt[4]{(1+N)(1-a)} \sqrt{\frac{R_S}{2D}}$$



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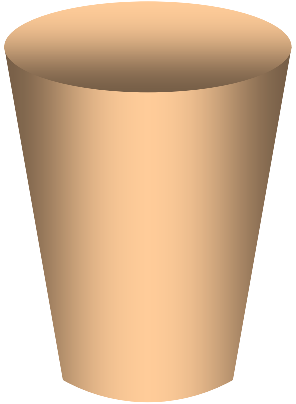
Building the actual object



Le verre répond-il au but visé?
Y entre-t-il bien 250 ml de café?
Que faudrait-il corriger?
Et si on voulait une capacité de 500 ml?