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Monad measure spaces and combinatorial number theory

A common theme in combinatorial number theory is to deduce structure in subsets of the natural numbers that are not small with respect to some density. Perhaps the most famous example of such a result is *Szemeredi's Theorem*, which states that if $A \subseteq \mathbb{N}$ has positive upper density, then A contains arbitrarily long arithmetic progressions.

A motivating example for this talk will be a theorem of Renling Jin, which states that if $A, B \subseteq \mathbb{N}$ both have positive Banach density, then $A+B$ is *piecewise syndetic*, meaning that there is a natural number k such that $A+B+[0,k]$ contains arbitrarily long intervals. Jin's proof uses *nonstandard analysis*, and, in particular, the notion of *Loeb measure*.

In this talk, we will focus on recent applications of nonstandard analysis to combinatorial number theory which rely not on the Loeb measure spaces, but rather on certain quotients of them called *monad measure spaces*. After defining the monad measure spaces, we will show how a Lebesgue Density Theorem for these spaces easily yields Jin's theorem. In addition, I will explain how, with a little more effort, one can even deduce certain quantitative versions of Jin's theorem.

I will end the talk with recent applications of a multiplicative (or logarithmic) version of the monad measure space construction, which we use to obtain approximate geo-arithmetic structure in sets of positive logarithmic density.

Much of the work presented in this talk is joint work with Mauro di Nasso, Renling Jin, Steven Leth, Martino Lupini, and Karl Mahlburg.