# THE 1989 ASIAN PACIFIC MATHEMATICAL OLYMPIAD

*Time allowed: 4 hours*
*NO calculators are to be used.*
*Each question is worth seven points.*

## Question 1

Let $x_1$, $x_2$, ..., $x_n$ be positive real numbers, and let

$$S = x_1 + x_2 + \cdots + x_n.$$

Prove that

$$(1 + x_1)(1 + x_2) \cdots (1 + x_n) \leq 1 + S + \frac{S^2}{2!} + \frac{S^3}{3!} + \cdots + \frac{S^n}{n!} .$$

## Question 2

Prove that the equation

$$6(6a^2 + 3b^2 + c^2) = 5n^2$$

has no solutions in integers except $a = b = c = n = 0$.

## Question 3

Let $A_1$, $A_2$, $A_3$ be three points in the plane, and for convenience, let $A_4 = A_1$, $A_5 = A_2$. For $n = 1$, 2, and 3, suppose that $B_n$ is the midpoint of $A_nA_{n+1}$, and suppose that $C_n$ is the midpoint of $A_nB_n$. Suppose that $A_nC_{n+1}$ and $B_nA_{n+2}$ meet at $D_n$, and that $A_nB_{n+1}$ and $C_nA_{n+2}$ meet at $E_n$. Calculate the ratio of the area of triangle $D_1D_2D_3$ to the area of triangle $E_1E_2E_3$.

## Question 4

Let $S$ be a set consisting of $m$ pairs $(a, b)$ of positive integers with the property that $1 \leq a < b \leq n$. Show that there are at least

$$4m \cdot \frac{(m - \frac{n^2}{4})}{3n}$$

triples $(a, b, c)$ such that $(a, b)$, $(a, c)$, and $(b, c)$ belong to S.

## Question 5

Determine all functions $f$ from the reals to the reals for which

(1) $f(x)$ is strictly increasing,
(2) $f(x) + g(x) = 2x$ for all real $x$,

where $g(x)$ is the composition inverse function to $f(x)$. (Note: $f$ and $g$ are said to be composition inverses if $f(g(x)) = x$ and $g(f(x)) = x$ for all real $x$.)