PROBLEMS FOR FEBRUARY

Please send your solutions to

E.J. Barbeau
Department of Mathematics
University of Toronto
40 St. George Street
Toronto, ON M5S 2E4

individually as you solve the problems. Electronic files can be sent to *barbeau@math.utoronto.ca*. However, please do not send scanned files; they use a lot of computer space, are often indistinct and can be difficult to download.

It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

661. Let P be an arbitrary interior point of an equilateral triangle ABC. Prove that

$$|\angle PAB - \angle PAC| \geq |\angle PBC - \angle PCB| \quad .$$

662. Let n be a positive integer and $x > 0$. Prove that

$$(1+x)^{n+1} \geq \frac{(n+1)^{n+1}}{n^n} x \quad .$$

663. Find all functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ such that

$$x^2 y2(f(x+y) - f(x) - f(y)) = 3(x+y)f(x)f(y)$$

for all real numbers x and y.

664. The real numbers x, y, and z satisfy the system of equations

$$x2 - x = yz + 1;$$

$$y2 - y = xz + 1;$$

$$z2 - z = xy + 1.$$

Find all solutions (x, y, z) of the system and determine all possible values of $xy + yz + zx + x + y + z$ where (x, y, z) is a solution of the system.

665. Let $f(x) = x3 + ax2 + bx + b$. Determine all integer pairs (a, b) for which $f(x)$ is the product of three linear factors with integer coefficients.

666. Assume that a face S of a convex polyhedron $\mathfrak{P}$ has a common edge with every other face of $\mathfrak{P}$. Show that there exists a simple (nonintersecting) closed (not necessarily planar) polygon that consists of edges of $\mathfrak{P}$ and passes through all the vertices.

667. Let A_n be the set of mappings $f : \{1, 2, 3, \cdots, n\} \longrightarrow \{1, 2, 3, \cdots, n\}$ such that, if $f(k) = i$ for some i, then f also assumes all the values $1, 2, \cdots, i-1$. Prove that the number of elements of A_n is $\sum_{k=0}^{\infty} k^n 2^{-(k+1)}$.