## PROBLEMS FOR JANUARY

Please send your solutions to

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no later than February 5, 2009.

Electronic files can be sent to *valeria.pandelieva@sympatico.ca*. However, if you respond electronically, please do not scan a handwritten solution. The attachment uses an inordinate amount of space and often causes difficulties in downloading. Also, the handwriting is frequently indistinct or splotchy. Please type the solution using a word-processing package (TeX is good and can be sent in a pdf file). If your file is not downloadable, then you will be asked to send your solutions by mail; do not use anything fancy or exotic.

It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

591. The point $O$ is arbitrarily selected from the interior of the angle $KAM$. A line $g$ is constructed through the point $O$, intersecting the ray $AK$ at the point $B$ and the ray $AM$ at the point $C$. Prove that the value of the expression
$$\frac{1}{[AOB]} + \frac{1}{[AOC]}$$
does not depend on the choice of the line $g$. [*Note:* $[MNP]$ denotes the area of triangle $MNP$.]

592. The incircle of the triangle $ABC$ is tangent to the sides $BC$, $CA$ and $AB$ at the respective points $D$, $E$ and $F$. Points $K$ from the line $DF$ and $L$ from the line $EF$ are such that $AK \| BL \| DE$. Prove that:

    (a) the points $A$, $E$, $F$ and $K$ are concyclic, and the points $B$, $D$, $F$ and $L$ are concyclic;

    (b) the points $C$, $K$ and $L$ are collinear.

593. Consider all natural numbers $M$ with the following properties:

    (i) the four rightmost digits of $M$ are 2008;

    (ii) for some natural numbers $p > 1$ and $n > 1$, $M = p^n$.

    Determine all numbers $n$ for which such numbers $M$ exist.

594. For each natural number $N$, denote by $S(N)$ the sum of the digits of $N$. Are there natural numbers $N$ which satisfy the condition severally:

    (a) $S(N) + S(N^2) = 2008$;

    (b) $S(N) + S(N^2) = 2009$?

595. What are the dimensions of the greatest $n \times n$ square chessboard for which it is possible to arrange 111 coins on its cells so that the numbers of coins on any two adjacent cells (*i.e.* that share a side) differ by 1?

596. A $12 \times 12$ square array is composed of unit squares. Three squares are removed from one of its major diagonals. Is it possible to cover completely the remaining part of the array by 47 rectangular tiles of size $1 \times 3$ without overlapping any of them?

597. Find all pairs of natural numbers $(x, y)$ that satisfy the equation
$$2x(xy - 2y - 3) = (x + y)(3x + y) \ .$$